



# The asymmetric spillover effect of the Markov switching mechanism from the futures market to the spot market

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## ABSTRACT

This article develops a multichain Markov switching dynamic conditional correlation ARCH model with idiosyncratic jump dynamics to investigate whether the state of the crude oil futures market can asymmetrically affect the state of the crude oil spot market. The asymmetric spillover effects are investigated after controlling the dependence structure on idiosyncratic jumps. The empirical findings show an asymmetric spillover effect from the futures market to the spot market. Moreover, the transition probabilities depend highly on the volatility of the futures market, showing the leading role of the futures market. The jump components play a relatively more important role in explaining the conditional variances than do the ARCH and regime-switching effects. Finally, both the contribution of idiosyncratic jumps on total variance and the correlation coefficient between the futures and spot returns rely on the volatility state of the futures and spot returns. The second and fourth moments of the conditional correlation coefficients will be underestimated when the common jumps and/or independent transition mechanisms are ignored. The findings of this paper have important implications for investors in accurately evaluating riskiness, hedgers in improving hedging performance, as well as market participants and government authorities in understanding the lead-lag relationship between crude oil spot and futures markets.

## 1. Introduction

The impact of the futures market on the spot market has been examined for many decades.<sup>1</sup> This paper proposes a multichain Markov switching dynamic conditional correlation ARCH model with idiosyncratic jump dynamics (MMSDCC-ARCH-ID-Jump model) to explore whether the futures market can influence the spot market. The aim of this paper is to investigate whether evidence can be obtained in support of a spillover effect in the Markov-switching process from the crude oil futures market to the crude oil spot market and a jump-interdependence effect between these two markets. Furthermore, this paper also aims to determine whether the jump-interdependence effect has a significant impact on the magnitude of the spillover effect.

Two of the most important factors that need to be considered in analyzing the dynamic volatility process of crude oil returns are regime shifts and irregular jumps. The importance of regime-switches in the volatility dynamics of crude oil futures and spot returns has been documented in several empirical studies (Wilson et al., 1996; Fong & See, 2002, 2003; Vo, 2009; Nomikos & Pouliasis, 2011, 2015; Ma et al., 2017; and; Zhang et al., 2019). For example, Wilson et al. (1996) observe that the variance of crude oil futures returns suffered

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<sup>1</sup> See Silvapulle and Moosa (1999) to review the reasons why futures markets can influence spot markets.

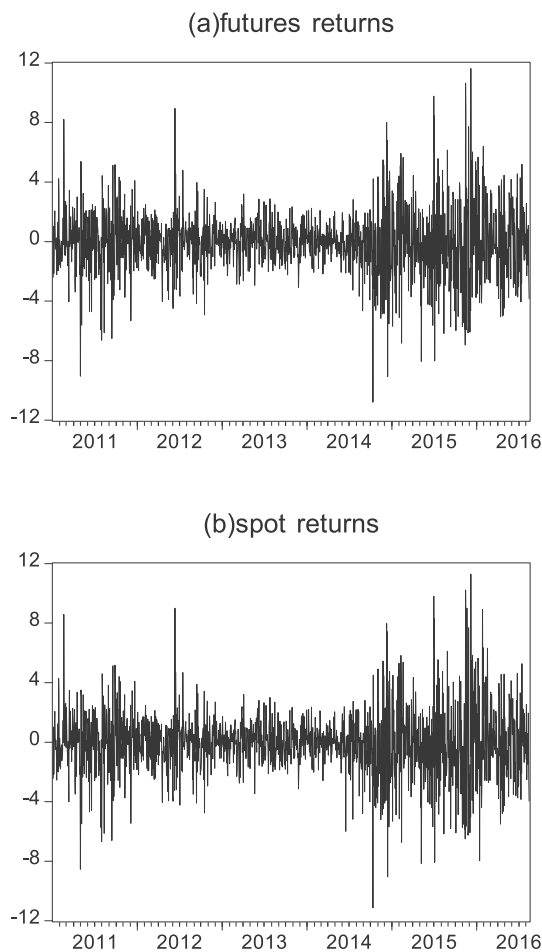


Fig. 1. Daily crude oil futures and spot returns.

15 structural changes over the period from January 1984 to December 1992, and that those changes reduced the volatility persistence. [Fong and See \(2002; 2003\)](#) utilize the Markov-switching GARCH model to analyze the temporal volatility of crude oil futures returns, and find that the temporal volatility process exhibits two dynamic patterns: a pattern of high volatility and one of low volatility. [Nomikos and Pouliasis \(2011\)](#) employ a mixture of GARCH model and regime-switching GARCH model to investigate the volatility process of petroleum futures returns. Their empirical results show evidence in support of structural change and volatility clustering. [Nomikos and Pouliasis \(2011\)](#) also investigate the term structural dynamics of petroleum futures markets by using a Markov-switching vector error correction model, and find that the volatility processes and disequilibrium adjustment processes differ in distinct states. The stochastic volatility model with a regime switching process is proposed by [Vo \(2009\)](#) to forecast the volatility of crude oil futures returns. The benefit of considering Markov switching for volatility prediction is that it greatly improves prediction accuracy. In order to allow the realized volatility model to switch under different volatility conditions, [Ma et al. \(2017\)](#) combine the regime switching property with a realized volatility process to analyze the volatility behavior of crude oil futures returns. The high level of persistence in the volatility process is reduced substantially when the regime switching property is considered. [Zhang et al. \(2019\)](#) find that the Markov-switching GARCH model has superior in-sample estimation accuracy compared to single-regime GARCH models.

In addition to regime shifts, the second factor requiring consideration in the analysis of crude oil returns is the irregular price jumps. [Lee et al. \(2010\)](#) utilize a component GARCH with a dynamic jump process and structural breaks to investigate the crude oil futures and spot returns for the period from 1990 to 2007. They observe a structural change in each return series, but with different breakpoints. The structural break point was on the 10th of November, 1999 for the crude oil spot market, and on the 17th of September, 2004 for the crude oil futures market. The jump behaviors showed some differences. [Lee et al. \(2010\)](#) consider that these jumps can be explained by the occurrence of sociopolitical and socioeconomic events, such as Iraq's invasion of Kuwait on 2 August 1990, the Operation Desert

**Table 1**  
Summary statistics.

	Daily futures returns	Daily spot returns
Mean	$-4.572 \times 10^{-2}$	$-4.572 \times 10^{-2}$
Median	-0.021	-0.009
Standard deviation	2.186	2.222
Minimum	-10.794	-11.126
Maximum	11.621	11.289
Skewness	0.115	0.151
Kurtosis	5.965	6.034
Jarque-Bera statistic	539.882***	567.619***
ADF unit root test	-41.218***	-40.714***
KPSS unit root test	0.112	0.122

Notes: \*\*\* indicates significance at the 1% level. The Jarque-Bera test statistic tests whether the returns follow a normal distribution. The ADF and KPSS unit root tests are used to test whether the return is stationary or non-stationary. The null hypothesis of ADF test is that the return series has a unit root. The null hypothesis of DPSS test is that the return series is stationary.

Storm beginning on 17 January 1991, the terrorist attacks on 11 September 2001, and the Iraq war beginning on 20 March 2003. Notably, the conditional variance in their experiment was partly explained by jumps. The ratio of variance in jump on the total variance was about 12.31% for the spot market and 21.85% for the futures market.<sup>2</sup> On the other hand, [Chen et al. \(2020\)](#) forecast the crude oil futures volatility based on the realized volatility model with jump and time-varying volatility. They observe that the empirical models with jumps and high persistence in volatility process have better out-of-sample forecasting performance.

A great number of empirical studies ([Foster, 1996](#); [Silvapulle & Moosa, 1999](#); [Bekiros & Diks, 2008](#); [Lee & Zeng, 2011](#); [Silverio & Szklo, 2012](#); [Alzahrani et al., 2014](#); [Chen et al., 2014](#); [Shrestha, 2014](#); [Balcilar et al., 2015](#); and; [Chang & Lee, 2015](#)) have investigated the relationship between crude oil futures and spot markets.<sup>3</sup> Unfortunately, no consistent evidence on the impact of the crude oil futures market on the crude oil spot market has yet been found.

The relationship between spot and futures markets is notably influenced by the time-varying volatility as shown by [Silvapulle and Moosa \(1999\)](#) and [Bekiros and Diks \(2008\)](#), who observe that the GARCH-type effect can be identified in the relationship between the crude oil spot and futures markets. Ignoring the volatility clustering property will result in overestimating the magnitude of their relationship. [Chan and Young \(2006\)](#) explore the joint dynamic between the copper spot and futures returns by proposing a bivariate GARCH model with common jump dynamics. Their empirical results indicate that not only the traditional GARCH effect, but also the common jump effect, is important characteristics of the joint dynamics. That is, the volatility clustering process with jump risk is a more suitable specification for capturing the time-varying conditional variance.

[Chen et al. \(2014\)](#) observe that structural change plays a decisive role in determining the cointegration relationship between crude oil futures and spot prices. In their study, the long-run relationship was not constant over the sample period, from January 1986 to December 2012. More specifically, there was a structural change in the long-run relationship in August 2004. [Alizadeh et al. \(2008\)](#) develop a bivariate Markov-switching GARCH model to calculate the hedging ratio and hedging performance for energy commodities. In their study, the same regime-switching mechanism for spot and futures markets is assumed. They document that the variance for each pair of energy commodities exhibits high-volatility and low-volatility states, and that the Markov-switching process is very important in enhancing the hedging performance. [Sheu and Lee \(2014\)](#) extend the multichain Markov regime switching of [Otranto \(2005\)](#) to include the GARCH effect in the conditional variance processes in order to examine the dynamic behavior between the spot and futures markets for many energy-related products, and investigate the hedging ability of the extended model. In contrast to the common Markov-switching mechanism of [Alizadeh et al. \(2008\)](#), [Sheu and Lee's \(2014\)](#) specification supposes that the state variables for the spot and futures markets are distinct. They found that each return series has high- and low-volatility states, and that the dynamic behaviors in each state differ. Furthermore, the multichain Markov regime switching GARCH model has better hedging gain compared to the bivariate GARCH model without the Markov-switching property.

Although some empirical studies have attempted to take the jumps or Markov-switching effects into account when exploring the

<sup>2</sup> The importance of jumps is also observed in the energy-related spot markets. For example, [Mason and Wilmot \(2014\)](#) explore whether the natural gas spot markets traded in the U.S. and U.K. exhibit jump effects; they find that the jump effect is a vital factor in the volatility of natural gas markets. The jump probabilities for the U.S. and U.K. markets are not always identical, and the intensity of the jump is stronger in the U.K. market than in the U.S. market.

<sup>3</sup> In addition to the crude oil market, commodity markets and financial markets have also been analyzed in extant empirical studies. For example, [Garbada and Silber \(1983\)](#) analyze the price-discovery effect for commodity markets, including wheat, corn, oats, orange juice, gold, and silver. [Figuerola-Ferretti and Gonzalo \(2010\)](#) discuss the mechanism of price discovery in the spot and forward markets, including aluminum, copper, nickel, lead, and zinc. Furthermore, [Judge and Reancharoen \(2014\)](#) provide an overview of empirical studies on various financial assets. [Hou and Li \(2020\)](#) investigate the relationship between China stock index and stock futures index futures markets during the period of stock market crisis of 2015 in terms of the volatility spillover and skewness spillover.

**Table 2**  
Estimation results.

Parameter	Model A	Model B	Model C	Model D
$\alpha_{10,0}$	-0.077 (0.051)	-0.073 (0.050)	-0.101** (0.050)	-0.079 (0.051)
$\alpha_{10,1}$	-0.062 (0.051)	-0.059 (0.050)	-0.116** (0.051)	-0.097* (0.056)
$\alpha_{11,0}$	-0.170*** (0.015)	-0.167*** (0.014)	-0.146*** (0.015)	-0.136*** (0.014)
$\alpha_{11,1}$	-0.158*** (0.014)	-0.158*** (0.014)	-0.155*** (0.017)	-0.161*** (0.017)
$\alpha_{20,0}$	-0.062 (0.051)	-0.059 (0.050)	-0.101** (0.050)	-0.077 (0.051)
$\alpha_{20,1}$	-0.057 (0.056)	-0.056 (0.054)	-0.026 (0.133)	-0.091* (0.054)
$\alpha_{21,0}$	-0.159*** (0.014)	-0.158*** (0.014)	-0.146*** (0.015)	-0.138*** (0.014)
$\alpha_{21,1}$	-0.159*** (0.017)	-0.158*** (0.016)	-0.066 (0.052)	-0.146*** (0.016)
$\alpha_{10,0}$	3.820*** (0.120)	3.793*** (0.117)	0.631*** (0.134)	0.914*** (0.178)
$\alpha_{10,1}$	3.829*** (0.109)	3.823 (0.108)	0.524*** (0.141)	1.254*** (0.194)
$\alpha_{11,0}$	0.176*** (0.019)	0.179*** (0.018)	0.132*** (0.018)	0.136*** (0.018)
$\alpha_{11,1}$	0.171*** (0.017)	0.174*** (0.018)	0.151*** (0.025)	0.114*** (0.020)
$\mu_1$			-0.101 (0.068)	-0.082 (0.085)
$\delta_1^2$			2.733*** (0.388)	3.319*** (0.543)
$\alpha_{20,0}$	3.827*** (0.107)	3.817*** (0.107)	0.627*** (0.133)	0.907*** (0.177)
$\alpha_{20,1}$	4.257*** (0.162)	4.215*** (0.156)	2.799*** (0.661)	1.159*** (0.192)
$\alpha_{21,0}$	0.167*** (0.017)	0.170*** (0.017)	0.130*** (0.018)	0.134*** (0.019)
$\alpha_{21,1}$	0.117*** (0.020)	0.125*** (0.020)	0.218* (0.116)	0.108*** (0.019)
Parameter	Model A	Model B	Model C	Model D
$\mu_2$			-0.102 (0.068)	-0.081 (0.085)
$\delta_2^2$			2.738*** (0.388)	3.341*** (0.546)
$\lambda$			1.182*** (0.193)	0.910*** (0.184)
$\rho_{00}$	0.998*** (0.327 × 10 <sup>-3</sup> )	0.998 (0.257 × 10 <sup>-3</sup> )	0.999*** (0.202 × 10 <sup>-3</sup> )	0.999*** (0.129 × 10 <sup>-3</sup> )
$\rho_{10}$	9.997 × 10 <sup>-1</sup> *** (0.028 × 10 <sup>-3</sup> )	9.997 × 10 <sup>-1</sup> *** (0.026 × 10 <sup>-3</sup> )	0.950*** (0.010)	0.126 (0.159)
$\rho_{01}$	0.726*** (0.031)	0.721*** (0.034)	0.375** (0.181)	0.954*** (0.008)
$\rho_{11}$	0.986*** (0.002)	0.986*** (0.002)	0.724** (0.312)	0.996*** (0.840 × 10 <sup>-3</sup> )
$\rho_j$			9.996 × 10 <sup>-1</sup> (0.082 × 10 <sup>-3</sup> )	9.996 × 10 <sup>-1</sup> *** (0.085 × 10 <sup>-3</sup> )
$\theta_{1,00}$	1.694*** (0.269)	1.695*** (0.260)	2.128*** (0.167)	4.387*** (0.363)
$\theta_{1,11}$	2.585*** (0.265)	2.442*** (0.252)	0.929*** (0.194)	3.177*** (0.326)
$\theta_{2,00}$	2.386*** (0.150)	2.847*** (0.362)	3.609*** (0.237)	2.245*** (0.170)
$\theta_{2,11}$	1.113*** (0.183)	0.895*** (0.345)	0.554* (0.283)	0.933*** (0.197)
$\theta_{2,02}$		-0.624 (0.459)		-1.424*** (0.388)
$\theta_{2,12}$		0.386 (0.439)		1.384*** (0.369)
Ln(L)	-2646.987	-2645.602	-2585.426	-2574.275
AIC	3.646	3.647	3.571	3.558
BIC	3.733	3.741	3.679	3.674

Notes: Figures in parentheses shown below the estimated coefficients are standard errors. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% significance levels, respectively. Equations (1)-(19) constitute the Model D (MMSDCC-ARCH-ID-Jump model). Model A is a simplified model of Model D by imposing the restriction that  $\mu_1 = \delta_1^2 = \mu_2 = \delta_2^2 = \lambda = \rho_j = \theta_{2,02} = \theta_{2,12} = 0$ . By imposing the restriction that  $\mu_1 = \delta_1^2 = \mu_2 = \delta_2^2 = \lambda = \rho_j = 0$ , the Model D reduces to Model B. The Model D becomes Model C when  $\theta_{2,02} = \theta_{2,12} = 0$ .

joint behavior of spot and futures returns, unfortunately, they do not consider the two effects simultaneously. The present study aims to fill this gap in the literature. In order to simultaneously consider both idiosyncratic jumps and the state dependent Markov switching mechanism, this study develops an MMSDCC-ARCH-ID-Jump model to investigate the potential impact of the futures return on the spot return. The potential spillover effects are evaluated after controlling the asymmetric time-varying and regime-dependent relationship. This differs from the works of Chan and Young (2006), Chan (2008, 2009), and Lee (2009), who employ the jump process to deal with the relationship between the futures and spot markets; these authors consider that both markets are simultaneously affected by the same jump process. Unfortunately, as the findings of Lee et al. (2010) have shown, the jump behaviors are not the same for crude oil futures and spot returns; hence, the specification of common jumps is not suitable for capturing their joint behavior. This paper does not use the common jump specification; instead it assumes that the crude oil futures and spot markets exhibit idiosyncratic jumps. While the design of the futures contract is based on its underlying spot asset, and the same shocks attributed to jumps affect both the futures and spot returns, the proposed specification allows the responses to jump shocks in the two markets to differ from each other. Moreover, the jump innovations for the two markets are assumed to correlate instead of existing independently.

In order to explore the potential impact of the futures market on the spot market, this paper employs a multichain Markov regime switching specification with volatility clustering effects. The impact of the futures market on the spot market is detected by examining whether there are spillover effects in Markov switching transition probabilities from the futures market to the spot market, after controlling for the idiosyncratic jump property. It is worth mentioning that, although Lee (2009) develops a regime switching GARCH model with conditional jump dynamics, his model adopts a common jump framework and hence ignores the possibility of distinct jump behaviors in different markets. Furthermore, although the empirical model put forward by Sheu and Lee (2014) allows for a multichain

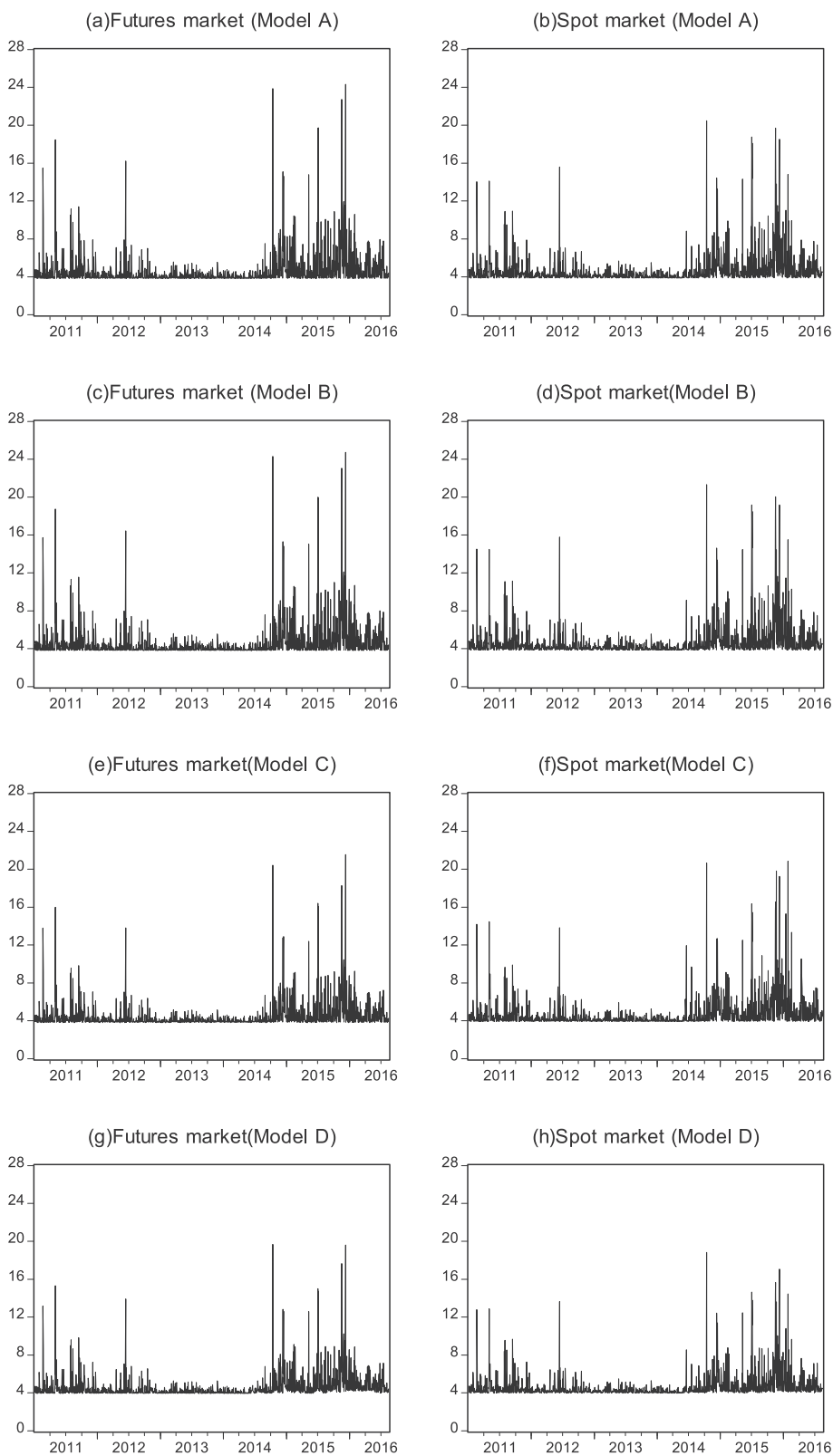


Fig. 2. Total conditional variances.

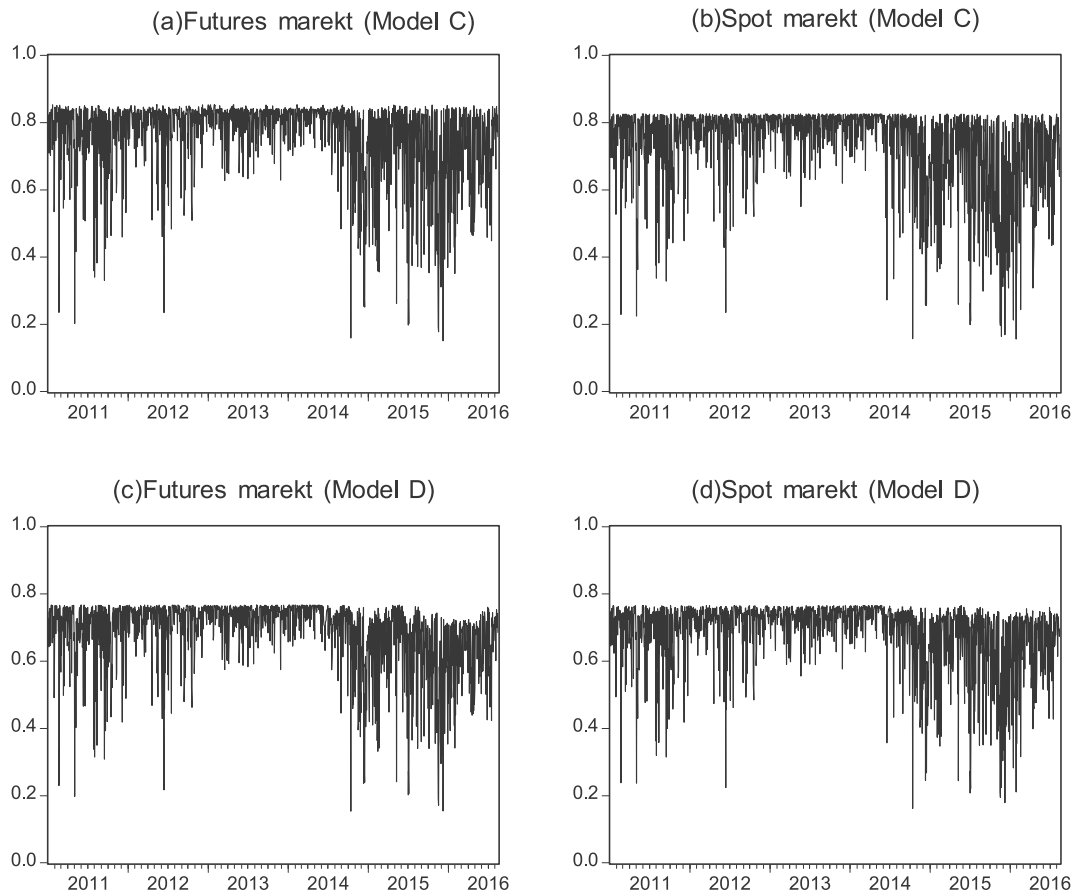


Fig. 3. The importance of jump risk to total conditional variance.

Markov switching mechanism, their model exhibits ignorance of jump effects, which is its insufficiency. To the best of our knowledge, this is the first study to consider idiosyncratic jumps in the analysis of the lead-lag relationship between the transition probabilities of crude oil futures and spot markets.

Four interesting topics are discussed in this paper. First, this paper examines the spillover effects of transition probabilities to determine whether the crude oil futures return can affect the crude oil spot return. Each return is assumed to have low-volatility and

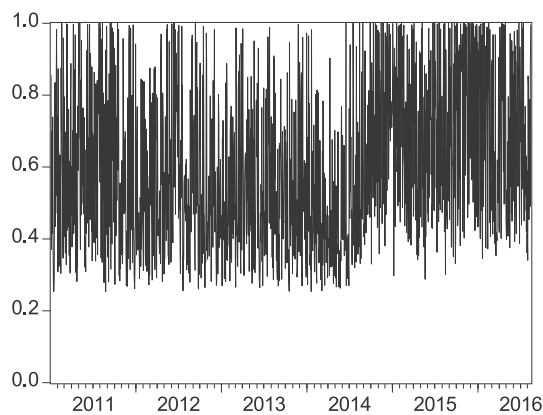
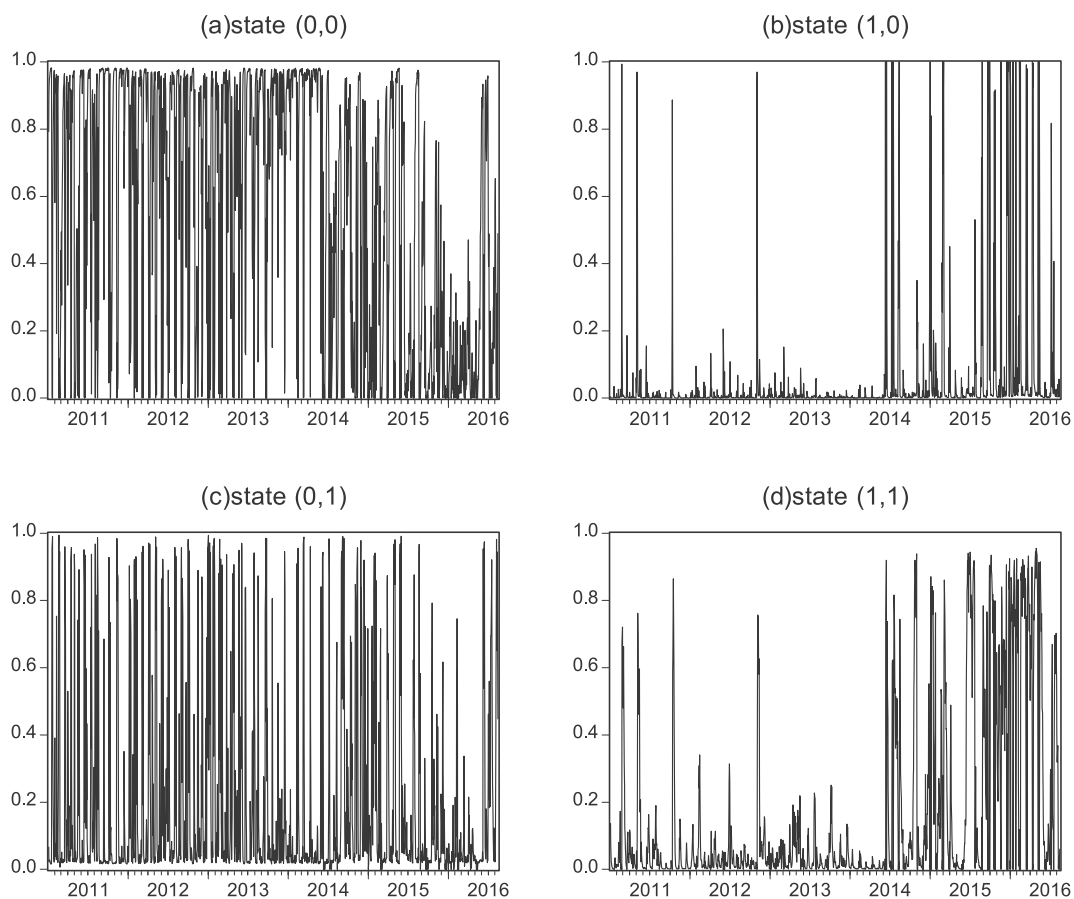


Fig. 4. The jump probabilities for Model D.

**Table 3**  
Transition probability matrices for Model D.

Panel A: Futures market	
$P_1 = \begin{bmatrix} 0.988 & 0.040 \\ 0.012 & 0.960 \end{bmatrix}$	
Panel B: Spot market	
$P_2^0 = \begin{bmatrix} 0.904 & 0.282 \\ 0.096 & 0.718 \end{bmatrix}$	$P_2^1 = \begin{bmatrix} 0.695 & 0.090 \\ 0.305 & 0.910 \end{bmatrix}$
Panel C: The joint matrix for futures and spot markets	
$P = \begin{bmatrix} 0.893 & 0.028 & 0.279 & 0.004 \\ 0.011 & 0.667 & 0.003 & 0.086 \\ 0.095 & 0.012 & 0.709 & 0.036 \\ 0.001 & 0.293 & 0.009 & 0.874 \end{bmatrix}$	

Notes:  $P_1$  is the transition probability matrix of futures market.  $P_2^0$  is the transition probability matrix of spot market, given the low-volatility state of futures market.  $P_2^1$  is the transition probability matrix of spot market, given the high-volatility state of futures market.  $P$  is the transition probability matrix for the four states. Model D is the MMSDCC-ARCH-ID-Jump model.



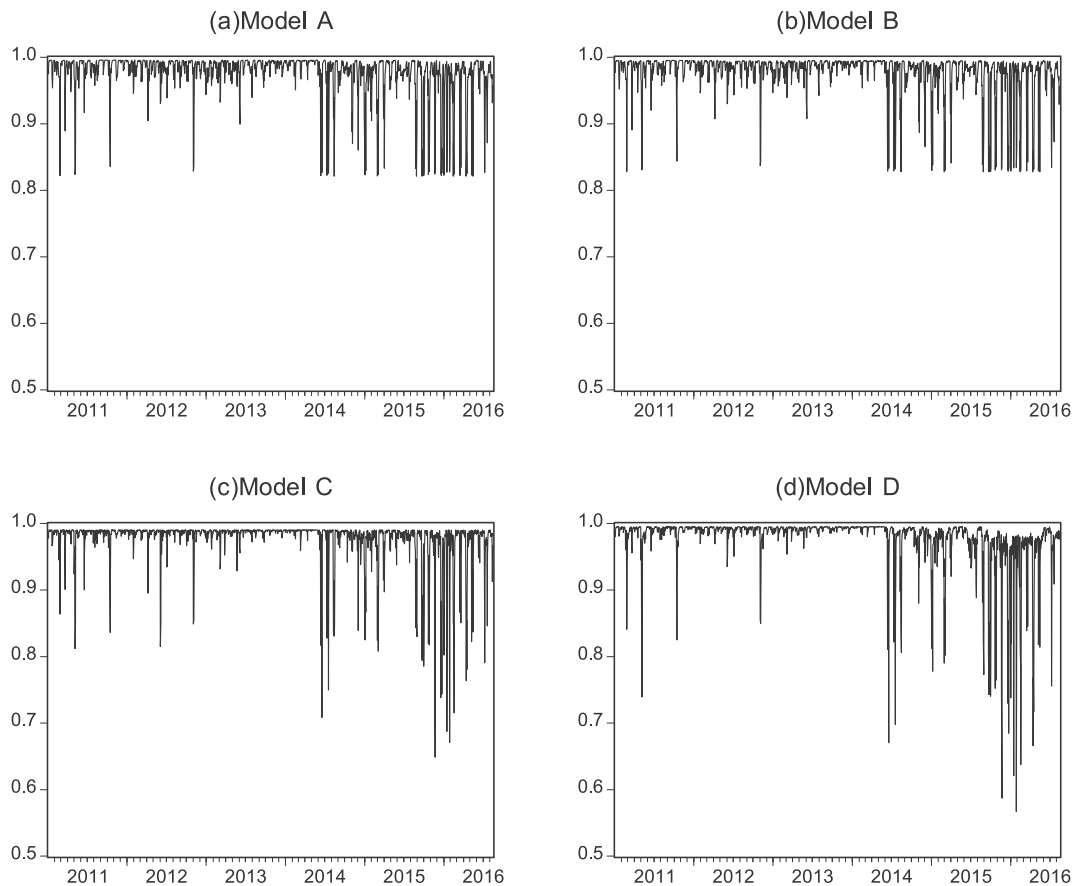
**Fig. 5.** Filtering probabilities for Model D.

high-volatility states. If today’s futures return can affect tomorrow’s spot return, then the spillover effects from the transition probabilities of the futures market to those of the spot market are documented. The relative magnitude of spillover effects in high-volatility and low-volatility states can then be compared. Second, this paper constructs an overall time-varying correlation coefficient based on both the state-dependent correlation coefficient between regular innovations of futures and spot returns and the jump-dependent

**Table 4**  
The proportion of each state in terms of filtering probabilities.

State	Model A	Model B	Model C	Model D
(0,0)	0.204	0.242	0.743	0.609
(1,0)	0.586	0.540	0.211	0.045
(0,1)	0.053	0.052	0.043	0.177
(1,1)	0.157	0.166	0.003	0.169

Notes: State (0,0) corresponds to low-volatility in both markets. State (1,1) corresponds to high-volatility in both markets. State (1,0) corresponds to high-volatility in the futures market but low-volatility in the spot market. State (0,1) corresponds to low-volatility in the futures market but high-volatility in the spot market. Equations (1)-(19) constitute the Model D (MMSDCC-ARCH-ID-Jump model). Model A is a simplified model of Model D by imposing the restriction that  $\mu_1 = \delta_1^2 = \mu_2 = \delta_2^2 = \lambda = \rho_j = \theta_{2,02} = \theta_{2,12} = 0$ . By imposing the restriction that  $\mu_1 = \delta_1^2 = \mu_2 = \delta_2^2 = \lambda = \rho_j = 0$ , the Model D reduces to Model B. The Model D becomes Model C when  $\theta_{2,02} = \theta_{2,12} = 0$ .



**Fig. 6.** Conditional correlation coefficients.

correlation coefficient. By assuming a time-varying dependence, this study could explore whether the relationship between futures and spot markets has distinct patterns in different time periods. Third, this paper analyzes whether the jump component can affect the conditional variance, and whether the importance of the jump component is different for the spot and futures markets. Fourth, this paper compares the proposed specification, which simultaneously considers the dependent Markov-switching mechanisms and idiosyncratic jumps, to specifications that do not consider these characteristics or consider only one of the two characteristics, in order to determine which specification can best improve the fitting performance and hedging performance.

The remainder of this paper is organized as follows. Section 2 introduces the MMSDCC-ARCH-ID-Jump model and constructs the conditional correlation coefficient. Section 3 reports the empirical results, including the identification of state variables, time-varying correlations, and the spillover effect of the transition probabilities. Three simplified models are examined in order to explore the importance of idiosyncratic jumps and state-dependence transition probabilities. Finally, Section 4 presents the conclusions.



## 2. The MMSDCC-ARCH-ID-Jump model

In order to allow for more flexible dynamics in the crude oil futures and spot markets, this study incorporates the dependent Markov-switching mechanisms and idiosyncratic jumps into a bivariate Markov-switching vector autoregressive model. The MMSDCC-ARCH-ID-Jump model is an extended version of the multichain Markov regime switching model put forward by [Otranto \(2005\)](#) and the common jump specification developed by [Chan and Young \(2006\)](#). This model is composed of three parts: mean processes, variance processes, and dependent transition probability matrices.

The mean equations follow a bivariate Markov-switching first-order autoregressive process with usual dependent innovations and idiosyncratic jump innovations expressed as follows:

$$r_{1t} = a_{10,s_{1t}} + a_{11,s_{1t}}r_{1,t-1} + \varepsilon_{1t,s_{1t}} + J_{1t} \tag{1}$$

$$r_{2t} = a_{20,s_{2t}} + a_{21,s_{2t}}r_{2,t-1} + \varepsilon_{2t,s_{2t}} + J_{2t} \tag{2}$$

where  $r_{1t}$  is the crude oil futures return,  $r_{2t}$  is the crude oil spot return,  $a_{10,s_{1t}}$  and  $a_{11,s_{1t}}$  are, respectively, the intercept term and autoregressive term for the futures return process,  $\varepsilon_{1t,s_{1t}}$  is the usual shock for the futures return,  $J_{1t}$  is the jump shock for the futures return,  $s_{1t}$  is a state variable corresponding to the futures return,  $a_{20,s_{2t}}$  and  $a_{21,s_{2t}}$  are, respectively, the intercept term and autoregressive term for the spot return process,  $\varepsilon_{2t,s_{2t}}$  is the usual shock for the spot return,  $J_{2t}$  is the jump shock for the spot return, and  $s_{2t}$  is a state variable corresponding to the spot return.

Obviously, there are two state variables,  $s_{1t}$  and  $s_{2t}$ , in the bivariate Markov-switching first-order autoregressive process. The three terms,  $a_{10,s_{1t}}$ ,  $a_{11,s_{1t}}$ , and  $\varepsilon_{1t,s_{1t}}$ , depend on the state variable  $s_{1t}$ ; similarly, the three terms,  $a_{20,s_{2t}}$ ,  $a_{21,s_{2t}}$ , and  $\varepsilon_{2t,s_{2t}}$ , depend on the state variable  $s_{2t}$ . This specification allows the evolution processes of the futures return and spot return to differ.

There are four error terms in Equations (1) and (2). The variance equations depict the dynamic processes for the four error terms. The terms  $\varepsilon_{1t,s_{1t}}$  and  $\varepsilon_{2t,s_{2t}}$  are related to each other, and  $J_{1t}$  is related to  $J_{2t}$ . However,  $\varepsilon_{it,s_{it}}$  is irrespective of  $J_{1t}$  and  $J_{2t}$ , and  $J_{it}$  is independent of  $\varepsilon_{1t,s_{1t}}$  and  $\varepsilon_{2t,s_{2t}}$ . The joint process for  $\varepsilon_{1t,s_{1t}}$  and  $\varepsilon_{2t,s_{2t}}$ , given two state variables and the information set  $\Omega_{t-1}$ , is assumed to have a bivariate normal distribution with the following form:

$$\begin{bmatrix} \varepsilon_{1t,s_{1t}} \\ \varepsilon_{2t,s_{2t}} \end{bmatrix} \Big| s_{1t}, s_{2t}, \Omega_{t-1} \sim BN \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} h_{1t,s_{1t}} & \rho_{s_{1t}s_{2t}} \sqrt{h_{1t,s_{1t}}h_{2t,s_{2t}}} \\ \rho_{s_{1t}s_{2t}} \sqrt{h_{1t,s_{1t}}h_{2t,s_{2t}}} & h_{2t,s_{2t}} \end{bmatrix} \right) \tag{3}$$

where  $\rho_{s_{1t}s_{2t}}$  is the state-dependent correlation coefficient between  $\varepsilon_{1t,s_{1t}}$  and  $\varepsilon_{2t,s_{2t}}$  and lies between  $-1$  and  $1$ . The terms  $h_{1t,s_{1t}}$  and  $h_{2t,s_{2t}}$  are the conditional variances for  $\varepsilon_{1t,s_{1t}}$  and  $\varepsilon_{2t,s_{2t}}$ , respectively, and have the following ARCH(1) processes.

$$h_{1t,s_{1t}} = \alpha_{10,s_{1t}} + \alpha_{11,s_{1t}}\varepsilon_{1,t-1}^2 \tag{4}$$

$$h_{2t,s_{2t}} = \alpha_{20,s_{2t}} + \alpha_{21,s_{2t}}\varepsilon_{2,t-1}^2 \tag{5}$$

where  $\alpha_{10,s_{1t}} > 0$ ,  $\alpha_{20,s_{2t}} > 0$ ,  $\alpha_{11,s_{1t}} \geq 0$ , and  $\alpha_{21,s_{2t}} \geq 0$ . Following the concept introduced by [Gray \(1996\)](#), we define  $\varepsilon_{1,t-1}^2$  and  $\varepsilon_{2,t-1}^2$  as shown below:

$$\varepsilon_{1,t-1} = r_{1,t-1} - E(r_{1,t-1} | \Omega_{t-2}) \tag{6}$$

$$\varepsilon_{2,t-1} = r_{2,t-1} - E(r_{2,t-1} | \Omega_{t-2}) \tag{7}$$

It should be strongly emphasized that there is a great difference between regime-dependent distribution and regime-independent distribution. According to the principle of [Has et al. \(2004\)](#),<sup>4</sup> the conditional joint distribution is a mixture of regime-dependent bivariate normal distributions with joint transition probabilities as the weighting mechanism. Hence, the joint distribution may be asymmetric, but not a bivariate normal distribution.

The jump processes are assumed to be irrelevant to the two state variables because jumps are temporary and sudden. The two jump-related error terms are expressed, respectively, as follows:

$$J_{1t} = \sum_{k=1}^n Y_{1t,k} - E \left[ \sum_{k=1}^n Y_{1t,k} \Big| \Omega_{t-1} \right] \tag{8}$$

<sup>4</sup> In the univariate framework, [Haas et al. \(2004\)](#) proved that the unconditional higher-order moments may totally differ from the traditional normal distribution when the regime-dependent conditional distribution follows a univariate normal distribution. The distribution given the information set  $\Omega_{t-1}$  is a mixture distribution of regime-dependent normal distributions with transition probabilities as the weighting mechanism. This means that the mixture distribution may account for the skewed property.

$$J_{2t} = \sum_{m=1}^n Y_{2t,m} - E \left[ \sum_{m=1}^n Y_{2t,m} \middle| \Omega_{t-1} \right] \tag{9}$$

where  $Y_{1t,k}$  and  $Y_{2t,k}$  are jump sizes and they are, respectively, assumed to have normal distributions,  $Y_{1t,k} \sim N(\mu_1, \delta_1^2)$  and  $Y_{2t,k} \sim N(\mu_2, \delta_2^2)$ . The number of jumps is captured by parameter  $n$  and follows a Poisson random variable with mean  $\lambda$ . In Equations (8) and (9),  $Y_{1t,k}$  and  $Y_{2t,k}$  have the same number of jumps due to the fact that the futures price and its underlying spot price are, to a great extent, governed by the same messages. While the futures and spot markets respond to the same jump shocks, the jump processes are allowed to be different for the two markets, and the two jump sizes have a correlation coefficient  $\rho_j$ . When the jump number is equal to  $k$ , the conditional covariance between  $Y_{1t}$  and  $Y_{2t}$  is  $\rho_j \sqrt{(k\delta_1^2)(k\delta_2^2)}$ .

The joint behavior of the two state variables follows the multichain Markov regime switching process proposed by Otranto (2005). The conditional probability for  $s_{1t}$  and  $s_{2t}$  is given by:

$$\Pr[s_{1t}, s_{2t} | s_{1t-1}, s_{2t-1}] = \Pr[s_{1t} | s_{1t-1}] \times \Pr[s_{2t} | s_{1t-1}, s_{2t-1}] \tag{10}$$

The above specification indicates that  $s_{1t}$  follows a time-invariant first-order Markov chain, while  $s_{2t}$  obeys a time-variant first-order Markov chain. That is, the occurrence of  $s_{1t}$  is determined only by the realization of  $s_{1t-1}$ . In contrast, the realization of  $s_{2t}$  is determined not only by the realization of  $s_{2t-1}$ , but also by the outcome of  $s_{1t-1}$ . Obviously, the multichain Markov regime switching process collapses into the independent Markov switching process when the realization of  $s_{1t-1}$  can not affect the realization of  $s_{2t}$ .<sup>5</sup>

Because existing empirical studies (Fong & See, 2002, 2003; Vo, 2009; Nomikos & Pouliasis, 2011; and; Ma et al., 2017) observe that the dynamics of the crude oil market can be successfully governed by a Markov-switching mechanism with high-volatility and low-volatility states, this paper assumes that each state variable has two distinct states: 0 and 1. State 0 is defined as the low-volatility state and state 1 is defined as the high-volatility state. The transition probability matrices are defined as follows:

$$P_1 = \begin{bmatrix} \Pr[s_{1t} = 0 | s_{1t-1} = 0] & \Pr[s_{1t} = 0 | s_{1t-1} = 1] \\ \Pr[s_{1t} = 1 | s_{1t-1} = 0] & \Pr[s_{1t} = 1 | s_{1t-1} = 1] \end{bmatrix} \tag{11}$$

$$P_2^0 = \begin{bmatrix} \Pr[s_{2t} = 0 | s_{2t-1} = 0, s_{1t-1} = 0] & \Pr[s_{2t} = 0 | s_{2t-1} = 1, s_{1t-1} = 0] \\ \Pr[s_{2t} = 1 | s_{2t-1} = 0, s_{1t-1} = 0] & \Pr[s_{2t} = 1 | s_{2t-1} = 1, s_{1t-1} = 0] \end{bmatrix} \tag{12}$$

$$P_2^1 = \begin{bmatrix} \Pr[s_{2t} = 0 | s_{2t-1} = 0, s_{1t-1} = 1] & \Pr[s_{2t} = 0 | s_{2t-1} = 1, s_{1t-1} = 1] \\ \Pr[s_{2t} = 1 | s_{2t-1} = 0, s_{1t-1} = 1] & \Pr[s_{2t} = 1 | s_{2t-1} = 1, s_{1t-1} = 1] \end{bmatrix} \tag{13}$$

where

$$\Pr[s_{1t} = 0 | s_{1t-1} = 0] = \frac{\exp(\theta_{1,00})}{1 + \exp(\theta_{1,00})} \tag{14}$$

$$\Pr[s_{1t} = 1 | s_{1t-1} = 1] = \frac{\exp(\theta_{1,11})}{1 + \exp(\theta_{1,11})} \tag{15}$$

$$\Pr[s_{2t} = 0 | s_{2t-1} = 0, s_{1t-1} = 0] = \frac{\exp(\theta_{2,00})}{1 + \exp(\theta_{2,00})} \tag{16}$$

$$\Pr[s_{2t} = 1 | s_{2t-1} = 1, s_{1t-1} = 0] = \frac{\exp(\theta_{2,11})}{1 + \exp(\theta_{2,11})} \tag{17}$$

$$\Pr[s_{2t} = 0 | s_{2t-1} = 0, s_{1t-1} = 1] = \frac{\exp(\theta_{2,00} + \theta_{2,02})}{1 + \exp(\theta_{2,00} + \theta_{2,02})} \tag{18}$$

$$\Pr[s_{2t} = 1 | s_{2t-1} = 1, s_{1t-1} = 1] = \frac{\exp(\theta_{2,11} + \theta_{2,12})}{1 + \exp(\theta_{2,11} + \theta_{2,12})} \tag{19}$$

The transition probability of  $s_{2t}$  is time-varying and is a function of the realizations of  $s_{1t-1}$  and  $s_{2t-1}$ . If  $s_{1t-1}$  has a significant impact on the transition probability of  $s_{2t}$ , it exhibits a spillover effect from the futures market to the spot market. The parameters  $\theta_{2,02}$  and  $\theta_{2,12}$  capture the impact of the futures market on the spot market. The first parameter,  $\theta_{2,02}$ , measures the spillover effect from the high-volatility state of the futures market to the low-volatility state of the spot market. If  $\theta_{2,02}$  is negative, the probability that the spot market will continue to stay in the low-volatility state is reduced when the futures market is in the high-volatility state. The second

<sup>5</sup> The independent Markov switching process is specified as follows:  $\Pr[s_{1t}, s_{2t} | s_{1t-1}, s_{2t-1}] = \Pr[s_{1t} | s_{1t-1}] \times \Pr[s_{2t} | s_{2t-1}]$ .

parameter,  $\theta_{2,12}$ , measures the spillover effect from the high-volatility state of the futures market to the high-volatility state of the spot market. If  $\theta_{2,12}$  is positive, the probability that the spot market will continue to stay in the high-volatility state increases when the futures market is in the high-volatility state. Furthermore, the spillover effect is asymmetric when the parameters  $\theta_{2,02}$  and  $\theta_{2,12}$  differ from each other.

Obviously, there are four distinct states for the joint dynamics: the first state corresponds to low-volatility in both markets ( $s_{1t} = 0, s_{2t} = 0$ ); the second state corresponds to high-volatility in the futures market but low-volatility in the spot market ( $s_{1t} = 1, s_{2t} = 0$ ); the third state indicates low-volatility in the futures market but high-volatility in the spot market ( $s_{1t} = 0, s_{2t} = 1$ ); and the fourth state indicates high-volatility in both markets ( $s_{1t} = 1, s_{2t} = 1$ ). The transition probability matrix for the four states can be expressed as follows:

$$P = P_1^* \odot P_2^* = (I_2 \otimes P_1) \odot \left( P_2^{0*} [., 1] \tilde{P}_2^{1*} [., 1] \tilde{P}_2^{0*} [., 2] \tilde{P}_2^{1*} [., 2] \right) \tag{20}$$

where  $\otimes$  is a Kronecker product,  $\odot$  is the element-by-element multiplication,  $P_1^j [., k]$  is the  $k$ th column of matrix  $P_1^j$ ,  $\tilde{\cdot}$  is an operator for concatenating column vectors horizontally,  $P_2^{0*} = P_2^0 \otimes \mathbf{1}_2, P_2^{1*} = P_2^1 \otimes \mathbf{1}_2$ , and  $\mathbf{1}_2$  is a  $2 \times 1$  column vector having all elements equal to 1.

The log-likelihood function has the following form:

$$\begin{aligned} \ln L &= \sum_{t=1}^T \ln f(r_{1t}, r_{2t} | \Omega_{t-1}) \\ &= \sum_{t=1}^T \left\{ \sum_{s_{1t}=0}^1 \sum_{s_{2t}=0}^1 \sum_{n=0}^{\infty} f(r_{1t}, r_{2t} | s_{1t}, s_{2t}, n, \Omega_{t-1}) \times P(n | \Omega_{t-1}) \times \right. \\ &\quad \left. P(s_{1t}, s_{2t} | \Omega_{t-1}) \right\} \end{aligned} \tag{21}$$

where  $P(s_{1t}, s_{2t} | \Omega_{t-1})$  is the predicting probability and can be obtained through the Bayesian iterative method employed in the traditional Markov-switching literature. Furthermore,  $f(r_{1t}, r_{2t} | s_{1t}, s_{2t}, n, \Omega_{t-1})$  is the conditional joint probability density function associated with two state variables and the number of jumps, whose form is expressed as follows:

$$\begin{aligned} f(r_{1t}, r_{2t} | s_{1t}, s_{2t}, n, \Omega_{t-1}) &= \frac{1}{2\pi \sqrt{(h_{1t,s_{1t}} + n\delta_1^2)(h_{2t,s_{2t}} + n\delta_2^2)} (1 - \rho_{dt,s_{1t},s_{2t}}^2)} \times \\ &\exp\left( -\frac{z_{1t,s_{1t}}^2 - 2\rho_{dt,s_{1t},s_{2t}} z_{1t,s_{1t}} z_{2t,s_{2t}} + z_{2t,s_{2t}}^2}{2(1 - \rho_{dt,s_{1t},s_{2t}}^2)} \right) \end{aligned} \tag{22}$$

where

$$z_{1t,s_{1t}} = \frac{r_{1t} - a_{10,s_{1t}} - a_{11,s_{1t}} r_{1,t-1} - n\mu_1 + \lambda\mu_1}{\sqrt{h_{1t,s_{1t}} + n\delta_1^2}} \tag{23}$$

$$z_{2t,s_{2t}} = \frac{r_{2t} - a_{20,s_{2t}} - a_{21,s_{2t}} r_{2,t-1} - n\mu_2 + \lambda\mu_2}{\sqrt{h_{2t,s_{2t}} + n\delta_2^2}} \tag{24}$$

$$\rho_{dt,s_{1t},s_{2t}} = \frac{Cov(r_{1t}, r_{2t} | s_{1t}, s_{2t}, n, \Omega_{t-1})}{\sqrt{Var(r_{1t} | s_{1t}, s_{2t}, n, \Omega_{t-1})} \sqrt{Var(r_{2t} | s_{1t}, s_{2t}, n, \Omega_{t-1})}} = \frac{\rho_{s_{1t},s_{2t}} \sqrt{h_{1t,s_{1t}}} \sqrt{h_{2t,s_{2t}}} + \rho_j \sqrt{(n\delta_1^2)(n\delta_2^2)}}{\sqrt{h_{1t,s_{1t}} + n\delta_1^2} \sqrt{h_{2t,s_{2t}} + n\delta_2^2}} \tag{25}$$

Finally, the time-varying correlation coefficient between the futures and spot returns depends on both the state-dependent

$$\begin{aligned} E[r_{1t} r_{2t} | \Omega_{t-1}] &= \sum_{s_{1t}=0}^1 \sum_{s_{2t}=0}^1 E[r_{1t} r_{2t} | s_{1t}, s_{2t}, \Omega_{t-1}] \times P(s_{1t}, s_{2t} | \Omega_{t-1}) \\ &= \sum_{s_{1t}=0}^1 \sum_{s_{2t}=0}^1 P(s_{1t}, s_{2t} | \Omega_{t-1}) \times \\ &\quad \left( \rho_{s_{1t},s_{2t}} \sqrt{h_{1t,s_{1t}}} \sqrt{h_{2t,s_{2t}}} + \rho_j \sqrt{(\mu_1^2 + \delta_1^2)\lambda} \sqrt{(\mu_2^2 + \delta_2^2)\lambda} \right) + \sum_{s_{1t}=0}^1 \sum_{s_{2t}=0}^1 P(s_{1t}, s_{2t} | \Omega_{t-1}) \times E[r_{1t} | s_{1t}, \Omega_{t-1}] \times E[r_{2t} | s_{2t}, \Omega_{t-1}] \end{aligned} \tag{27}$$

correlations and jump-dependent correlation, and is given by:

$$\rho_t = \frac{Cov(r_{1t}, r_{2t} | \Omega_{t-1})}{\sqrt{Var(r_{1t} | \Omega_{t-1})} \sqrt{Var(r_{2t} | \Omega_{t-1})}} = \frac{E[r_{1t} r_{2t} | \Omega_{t-1}] - E[r_{1t} | \Omega_{t-1}] E[r_{2t} | \Omega_{t-1}]}{\sqrt{Var(r_{1t} | \Omega_{t-1})} \sqrt{Var(r_{2t} | \Omega_{t-1})}} \tag{26}$$

where

$$Var(r_{it} | \Omega_{t-1}) = \sum_{s_{1t}=0}^1 \sum_{s_{2t}=0}^1 P(s_{1t}, s_{2t} | \Omega_{t-1}) \times [h_{it,s_{it}} + (E(r_{it} | s_{1t}, s_{2t}, \Omega_{t-1}))^2] - (E(r_{it} | \Omega_{t-1}))^2 + (u_i^2 + \delta_i^2) \lambda, \quad i = \{1, 2\} \tag{28}$$

### 3. Data and estimation results

#### 3.1. Data

The daily crude oil spot and futures prices provided by the Energy Information Administration (EIA) are investigated in this paper. The spot price refers to the WTI Light Sweet crude oil price, and the futures price refers to the NYMEX WTI Light Sweet crude oil futures contract with the earliest delivery date. The sample period is from January 3, 2011 to October 31, 2016. The return is defined as the first-order difference of natural logarithms of price indices multiplied by 100. Fig. 1 shows the futures and spot returns. As seen in this figure, volatility clustering is apparent for each return series, and the two returns are relatively stable over the period from 2012 to 2013. The volatility patterns seem to change very frequently. The range of returns noticeably increases from 2014 onwards. Furthermore, there are sudden and large spikes in the return series, and these return spikes seem uncorrelated. Hence, structural changes, volatility clustering, and jumps appear to be very important factors in analyzing the joint behavior of the crude oil futures and spot returns.

The summary statistics are reported in Table 1. According to the testing results of the Jarque-Bera test statistic (Jarque & Bera, 1980), the futures and spot returns do not follow a normal distribution. Based on the ADF and KPSS unit root tests, evidence of stationarity is observed for each return series. In addition, the descriptive statistics for spot and futures returns show a slight difference. The median, standard deviation, skewness, and kurtosis are slightly larger in the spot market than in the futures market. The minimum value and maximum value for the spot returns are smaller than those for the future returns. These differences reveal that the dynamic processes for both futures and spot returns differ from each other.

#### 3.2. Estimation results

Equations (1)-(19) constitute the MMSDCC-ARCH-ID-Jump model. For comparison purpose, the MMSDCC-ARCH-ID-Jump model is hereafter called Model D. Furthermore, in order to emphasize the importance of the idiosyncratic jump dynamics and the multichain Markov switching dynamics, this paper considers three simplified models. Model A (IMSDCC-ARCH model) is an independent Markov switching dynamic conditional correlation ARCH model that does not consider the idiosyncratic jump effects and spillover effect in the transition probabilities. Specifically, Model A is a simplified model of MMSDCC-ARCH-ID-Jump model by imposing the restriction that  $\mu_1 = \delta_1^2 = \mu_2 = \delta_2^2 = \lambda = \rho_j = \theta_{2,02} = \theta_{2,12} = 0$ . Model B (MMSDCC-ARCH model) is a multichain Markov switching dynamic conditional correlation ARCH model that does not consider the idiosyncratic jump effects. That is, imposing the restriction that  $\mu_1 = \delta_1^2 = \mu_2 = \delta_2^2 = \lambda = \rho_j = 0$ , the MMSDCC-ARCH-ID-Jump model reduces to Model B. Finally, Model C (IMSDCC-ARCH-ID-Jump model) is an independent Markov switching dynamic conditional correlation ARCH model with idiosyncratic jump effects. Specifically, the MMSDCC-ARCH-ID-Jump model becomes Model C when  $\theta_{2,02} = \theta_{2,12} = 0$ . Obviously, Model D (MMSDCC-ARCH-ID-Jump model) nests the above three simplified models.

The empirical results are presented in Table 2. The AIC and BIC values are reported in the last two rows of Table 2. Model D (MMSDCC-ARCH-ID-Jump model) has the smallest AIC and BIC values; Model C has the second largest AIC and BIC values; and Model B has the largest AIC and BIC values. These results indicate that Model D (MMSDCC-ARCH-ID-Jump model) is the best model, revealing the importance of idiosyncratic jump dynamics and the multichain Markov switching dynamics.

The parameter estimates for Model D and their corresponding standard errors are reported in the last column of Table 2. The empirical results reveal that each of the futures and spot returns has two states; hence, there are four different states in the joint dynamics. Furthermore, the parameter estimates of the futures return differ slightly from those of the spot returns, signifying that the dynamic behaviors of the futures and spot returns are not completely identical. With respect to the parameters shown in the mean equations, the autoregressive terms are all negative for each state and each return series. In state 0 (low-volatility state), the autoregressive coefficient of the spot return is slightly smaller than that of the futures return. However, in state 1 (high-volatility state), the futures return has a smaller autoregressive coefficient than the spot return. These results support the distinct persistent patterns in the futures and spot returns.

Evidence of the state-dependent volatility clustering effect and jump effect is documented in both the futures and spot returns. For each market, the state-dependent variance attributed to the ARCH effect is larger in state 1 than in state 0. Furthermore, in each state the

variance is larger in the futures return than in the spot return. The state-dependent variance is about 1.058 in state 0 and 1.415 in state 1 for the futures return. For the spot return, the variance due to the ARCH effect is about 1.047 in state 0 and 1.299 in state 1. In addition, the spot return has slightly less volatility persistence in each state than the futures return. The persistence is stronger in state 0 than in state 1 for each return series.

The idiosyncratic jump effect is analyzed next. The mean of the jump size is negative but insignificant for both markets. The variance in jump sizes is slightly larger in the spot market than in the futures market: it is about 3.319 for the futures return and about 3.341 for the spot return. The correlation coefficient between the two jump sizes is very close to 1, attesting that the two jump sizes are affected by similar messages.

As can be seen from Equation (28), the total conditional variances can be divided into two components: the variation arising from ARCH and Markov-switching effects and the variation arising from the jump effect. Fig. 2 plots the total conditional variances for the four models. The volatility patterns are similar in each model. The conditional variances estimated by Model D are shown in the last row of Fig. 2. Similar volatility processes for the futures returns and spot returns are observed. Intense volatility is observed before 2012 and after 2014, but especially in the latter period when the intensity increases.

This paper investigates which component has a greater influence on the total conditional variance and whether this influence changes over time. The time-varying ratio of variance due to the jump effect to total variance is shown in Fig. 3. It is clearly evident that a large part of the total conditional variance can be explained by the jump effect for crude oil futures and spot returns. For Model C, the sample median of the ratio is about 0.808 in the futures market, and about 0.784 in the spot market. For Model D, the sample medians are, respectively, 0.720 and 0.724 in the futures market and spot market. Obviously, the contribution of the jump effect will be over-estimated when the dependent Markov switching dynamics is ignored. Furthermore, the contribution of the jump effect is time-varying. The importance of the jump effect is relatively smaller before 2012 and after 2014. When both markets are in the high-volatility state, the contribution of ARCH and Markov-switching effects to the total variance increases. Fig. 4 depicts the jump probabilities for Model D.<sup>6</sup> The jump probability refers to the probability that the jump number is greater than 0. The sample mean and sample median for the jump probabilities are 0.596 and 0.544, respectively.

This paper next discusses the spillover effects of the transition probabilities and state identifications. The conditional transition probability matrices are reported in Table 3. For the futures market, the estimate of  $\theta_{1,00}$  is larger than that of  $\theta_{1,11}$ , indicating that, for this market, the persistence of the low-volatility state is larger than that of the high-volatility state. The probability of staying in the low-volatility state during any two consecutive dates is 0.988, while the probability of staying in the high-volatility state is about 0.960.

On the other hand, the coefficients for the transition probabilities of the spot market are all positive, except for  $\theta_{2,02}$ . The estimates for  $\theta_{2,02}$  and  $\theta_{2,12}$  are statistically significantly different from 0, showing that the state of the futures market in the current period can affect the state of the spot market in the next period. However, in the transition probabilities, the spillover effects from the futures market to the spot market are closely connected to the volatility state of the futures market. In addition, the spillovers in the transition probabilities are asymmetric. When the futures market is in the low-volatility state, the probabilities that the spot market will remain in the low-volatility state and in the high-volatility state are 0.904 and 0.718, respectively. When the futures market is in the high-volatility state, the probabilities that the spot market will remain in the low-volatility state and in the high-volatility state are 0.695 and 0.910, respectively. More specifically, when the futures market is in the high-volatility state, the spillover effect of the futures market on the occurrence of a low-volatility spot market is negative; however, the spillover effect on the probability that the spot market will stay in a high-volatility state is positive. These findings support the evidence that the futures market has a significant impact on the spot market due to the first-order Markov transition mechanism, and that the spillover effects are asymmetric.

Finally, as shown in Panel C of Table 3, the joint transition matrix for the futures and spot markets exhibits different types of persistence. The persistence is highest when the two markets are in the low-volatility state. When both markets are in the high-volatility state, the persistence is the second largest. The persistence is lowest when the futures market is in the high-volatility state and the spot market is in the low-volatility state.

Fig. 5 shows the filtering probabilities for Model D.<sup>7</sup> The high-volatility state commonly occurred from 2014 to the end of the sample period. The classifications of state variables are reported in Table 4. Filtering probabilities were employed to identify the state in each time period. The state with the largest filtering probability was recognized as the true state. The proportions for each state differ among different empirical models. For example, the proportion of occurrence of a high-volatility state in both markets is 0.157 for Model A, 0.166 for Model B, 0.003 for Model C, and 0.169 for Model D. For Model D, the proportion of occurrence of a low-volatility state in both markets is about 0.609. During the whole sample period, the state of low-volatility in the futures and spot markets comprised the largest proportion of occurrences. Low-volatility in the futures market combined with high-volatility in the spot market comprised a greater proportion of occurrences than the converse situation (high-volatility in the futures market along with low-volatility in the spot market). The evidence shows that periods in which the two markets are in a high-volatility state will be underestimated when the dependent structures in the state variables and/or jump processes are ignored.

The state-dependent correlations for random errors related to the ARCH effect are higher than 0.950, except when the futures market is in the high-volatility state and the spot market is in the low-volatility state. The conditional correlation coefficients are depicted in Fig. 6. The correlation coefficients become smaller during periods when the two markets are in the high-volatility state. The sample mean for conditional correlation correlations is very high and close to one. Its value is about 0.976 for Model A and Model C. The sample mean is about 0.977 for Model B and Model C. Hence, the crude oil futures and spot returns have a very tight relationship and the

<sup>6</sup> The jump probability is the weighting sum of state-dependent jump probabilities with filtering probabilities as weights.

<sup>7</sup> The filtering probabilities for the remaining three models are available upon request.

correlation almost approaches 1, irrespective of the empirical models. However, the high-order moments for different models are very different. The standard deviation is largest in Model D. The standard deviation is 0.035 for Model A, 0.034 for Model B, 0.038 for Model C, and 0.045 for Model D. The skewness is negative for all the models; Model D has the smallest skewness.<sup>8</sup> The leptokurtosis is most severe in Model D as compared to the remaining three models.<sup>9</sup> These findings indicate that, when the idiosyncratic jumps and/or dependent transition probabilities are not considered in the empirical specification, the standard deviation, the absolute value of skewness, and kurtosis of the conditional correlation coefficients will be underestimated.

As mentioned above, the patterns of conditional correlation coefficients differ across different models; therefore, the hedging allocation and hedging performance will be greatly affected by the empirical specifications. To clarify this point, this paper further investigates how idiosyncratic jumps and dependent Markov-switching mechanisms affect the implementation of the minimum-variance hedging strategy.<sup>10</sup> The sample mean of minimum-variance hedge ratios is about 0.983 for Model A, 0.984 for Model B, 0.995 for Model C, and 0.979 for Model D. Model D has the smallest sample mean of hedge ratios, and Model C has the largest sample mean of hedge ratios. On the other hand, the daily average return of hedged portfolio is 0.215% for Model A, 0.195% for Model B, 0.162% for Model C, and 0.457% for Model D. Thus, Model D gives the best hedging performance as compared to the other three models. Obviously, specifications without idiosyncratic jumps and/or dependent Markov-switching mechanisms will result in a misallocation of hedged strategy, and subsequently a comparatively poor hedge performance.

#### 4. Conclusions

This paper investigates whether the crude oil futures market can affect the spot market and whether the impacts are asymmetric by proposing a MMSDCC-ARCH-ID-Jump model, which considers the Markov-switching property, jump process, and volatility clustering. Three important findings are observed. First, the crude oil futures and spot returns have different return dynamics, variance dynamics, and Markov transition matrices. The dynamic processes show two distinct patterns: a pattern of low volatility and one of high volatility. Second, there is significant evidence that the crude oil futures market can asymmetrically affect the crude oil spot market through transition probabilities, supporting the notion that the futures market leads the spot market. In sum, a highly volatile futures market pushes the spot market to switch from a low-volatility state to high-volatility state, thus increasing the magnitude of volatility and also increasing the probability of the spot market remaining in this high-volatility state. Third, the MMSDCC-ARCH-ID-Jump model yields the greatest hedging benefit compared to other competing models. When empirical specifications ignore the dependent Markov-switching property and/or idiosyncratic jump process, the hedging strategy will be misestimated and the hedging performance will be underestimated.

These findings reveal important information for investors, hedgers, market participants, and government authorities, all of whom want to better understand the relationship and behavior of crude oil futures and spot markets. Because the conditional dynamics differ for these two markets, gaining insight into their relationship can help investors more accurately calculate the value-at-risk of an investment in these markets. For firms with hedging demands, understanding the impact of the dependent Markov-switching mechanism and idiosyncratic jump process on hedging ratios can help improve the hedging performance. Furthermore, understanding that the crude oil futures market leads the crude oil spot market is very important for market participants and government authorities, who need to monitor and regulate the crude oil spot market. It is worth emphasizing that these findings are restricted to the crude oil futures and spot markets, and cannot be directly applied to commodity markets and financial markets. This is because that due to heterogeneity and idiosyncrasy, volatility patterns and periods of low-volatility and high-volatility states will differ in different markets. Exploring commodity markets and financial markets and comparing their differences in regime-switching behaviors deserves future investigation.

#### Credit author statement

Kuang-Liang Chang: Conceptualization; Methodology; Software; Validation; Formal analysis; Investigation; Writing - Original Draft; Funding acquisition; Data Curation.

Chingnum Lee: Methodology; Investigation; Writing - Review & Editing.

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<sup>8</sup> The skewness is  $-3.226$  for Model A,  $-3.266$  for Model B,  $-4.396$  for Model C, and  $-4.679$  for Model D.

<sup>9</sup> The kurtosis is 13.303 for Model A, 13.505 for Model B, 24.886 for Model C, and 28.371 for Model D.

<sup>10</sup> This paper reports in-sample hedging results for unconstrained minimum-variance hedge ratios. The minimum-variance hedge ratio is determined by the conditional correlation between futures and spot returns, the conditional variance of the futures return, and the conditional variance of the spot return. Full results are available upon request.

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