Behavior of the Standard Dicky-Fuller Test When There is a Fourier-Form Break Under the Null Hypothesis

Paper Presented at

Department of Public Finance, National Chengchi University

03/24/2017

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Motivation

- Why Unit Root Test is Important?

AR(1) Process: $Y_t = \emptyset Y_{t-1} + \varepsilon_t.$

By Recursive Substitution, We get

$$Y_t = \varepsilon_t + \emptyset\varepsilon_{t-1} + \emptyset^2\varepsilon_{t-2} + \emptyset^3\varepsilon_{t-3} + \cdots$$

When $\lvert\emptyset\rvert < 1$	When $\lvert\emptyset\rvert = 1$
$\Rightarrow$Var (Y_t) $\rightarrow$ k,	$\Rightarrow$Var (Y_t) $\rightarrow \infty$.
where k is a finite.	

Motivation

- Standard Unit Root Test: Dickey-Fuller (1979):

$$Y_t = \alpha + \gamma t + \emptyset Y_{t-1} + \varepsilon_t,$$

$$H_0 : \emptyset = 1 \quad v.s.$$

$$H_1 : \emptyset < 1. \qquad (1)$$

Standard Dickey-Fuller Unit Root Test is t-test statistics on OLS estimation of $\emptyset$.

Motivation

- Standard Unit Root Test: Dickey-Fuller (1979):

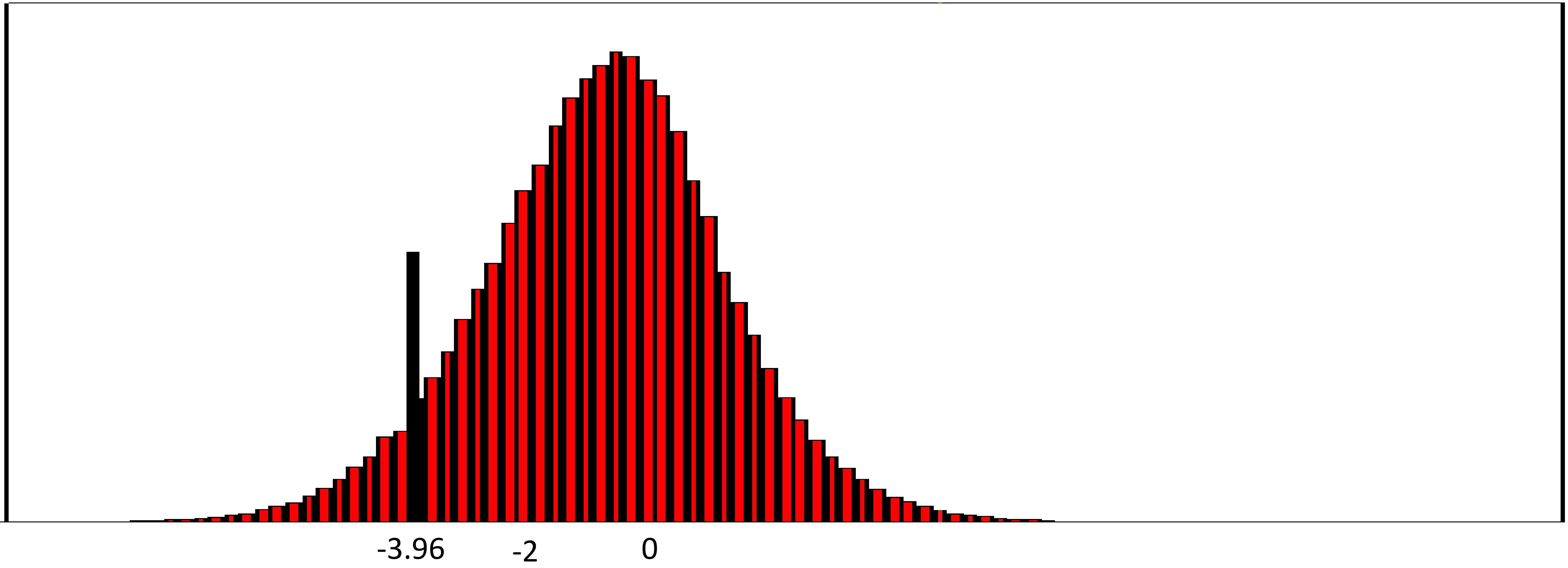
For example in a no constant and no trend model,

$$\check{t} = \frac{(\breve{\emptyset}_T - 1)}{\breve{\sigma}_{\breve{\emptyset}_T}} \xrightarrow{L} \frac{1/2\{[W(1)]^2 - 1\}}{\left\{\int_0^1 [W(r)]^2 dr\right\}^{1/2}}$$, (Dicky Fuller Distribution)

where $\breve{\sigma}^2_{\breve{\emptyset}T} = [s_T^2 \div \sum_{t=1}^T Y_{t-1}^2]^{1/2}$ and s_T^2 denote the OLS estimate of the disturbance variance: $s_T^2 = \sum_{t=1}^T \left(Y_t - \breve{\emptyset}_T Y_{t-1}\right)^2 / (T-1)$.

Motivation

- Standard Unit Root Test: Dickey-Fuller (1979):
  - Dicky Fuller Distribution (with constant and trend)



Motivation

- Standard Unit Root Test: Dickey-Fuller (1979):

$$Y_t = \alpha + \gamma t + \emptyset Y_{t-1} + \varepsilon_t,$$

$$H_0 : \emptyset = 1 \quad v.s.$$

$$H_1 : \emptyset < 1.$$

	True : $\emptyset = 1$	**True : $\emptyset < 1$**
Accept H_0	Correct	Type II Error (Low power)
Reject H_0	Type I Error (Size Distortion)	Correct

Schwart (1987) and Phillips-Perron (1988) :
DF had serious Size Distortion and Lack of Power

Motivation

Unit Root Tests and Structural Breaks

- ONE Exogenous Dummy Variable Structural Break Unit root test: Perron (1989)

$$y_t = \begin{cases} \alpha_1 + \gamma t + \emptyset Y_{t-1} + \varepsilon_t \;, for\; t < T_0 \\ \alpha_2 + \gamma t + \emptyset Y_{t-1} + \varepsilon_t \;, for\; t \geq T_0 \end{cases} \tag{2}$$

  - Perron Phenomenon (誤以為是單根，Power問題) : For $\emptyset < 1$ in (2),

    Perron (1989) found that with a significant shift and using standard DF tests one could **rarely reject the unit root** hypothesis, even in cases where we have a stationary process with a shift-mean.

  - Converse Perron Phenomenon（誤以為不是單根，Size問題）: For $\emptyset = 1$ in (2),

    Leybourn and Newbold (2000) find that if the true generating process is integrated of order one, but with a break, then it is shown that, if the break occurs early in the series, routine application of standard Dickey-Fuller tests can lead to a very serious problem of **spurious rejection of the unit root** null hypothesis.

Motivation

- Unit Root Tests and Structural Breaks
  - Subsequent Development after Perron (1989) .
    - Endogenous Structural Breaks:

      The main innovation of these papers is to suggest that the date of the break T_0 should be identified endogenously when testing for breaks. Ex: Banerjee, Lumsdaine and Stock (1992), and Zivot and Andrews (1992) etc.

    - Multiple break. Ex: Papell and Prodan (2003), and Kapetanois (2005)

Motivation

- Unit Root Tests and Structural Breaks
  - Prodan (2008) shows that it can be quite **difficult to properly estimate the number and the magnitudes of multiple breaks**.
  - Unit root with Fourier Change: Instead of adopting dummy variables to capture discrete breaks, several articles develop unit-root tests by applying Gallants (1981) **flexible Fourier form to take into account smoothing breaks** in the deterministic components (Becker, Enders and Hurn, 2004; Becker, Enders and Lee, 2006; Enders and Lee, 2012a,b; Rodrigues and Taylor, 2012; Lee et al. 2016)
    - Fourier Transformation to Approximate Break.

$$d(t) = \beta_0 + \sum_{k=1}^{n} \beta_{1k} \sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{n} \beta_{2k} \cos\left(\frac{2\pi kt}{T}\right),$$

where k is the frequency parameter.

Motivation

- Unit Root Tests and Structural Breaks

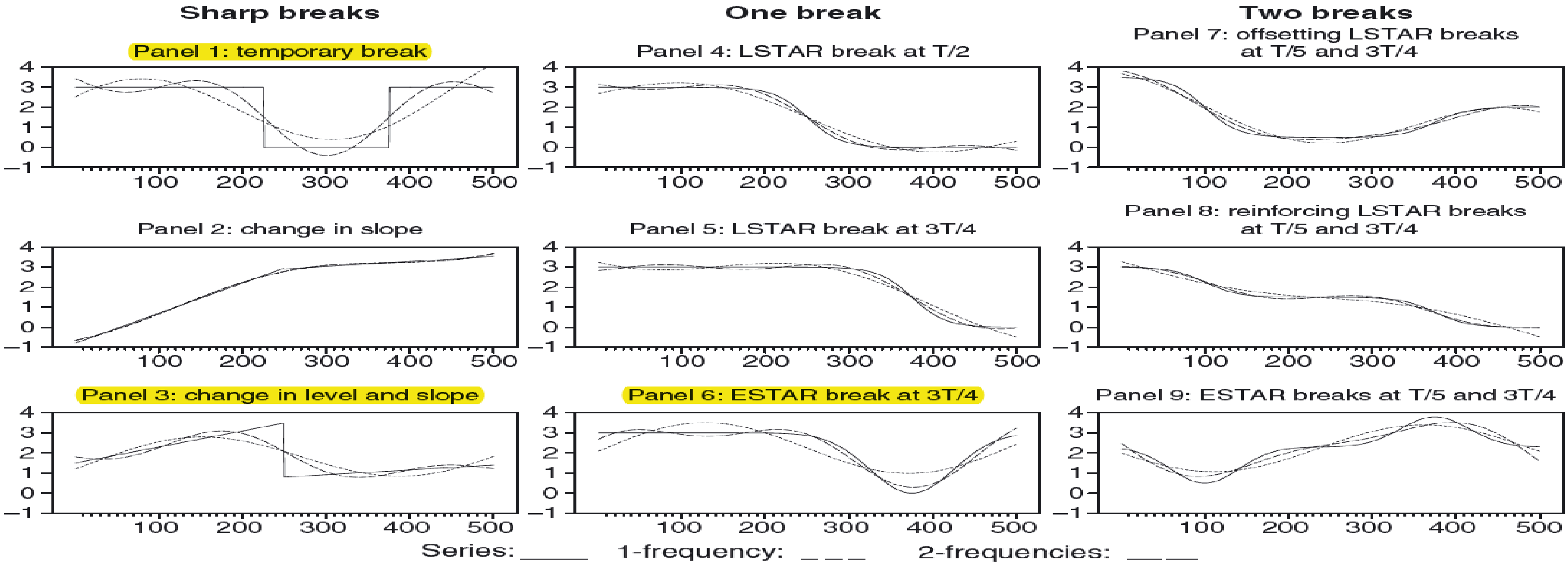


Figure 1. Sharp, ESTAR and LSTAR breaks

Motivation

Unit Root Tests and Structural Breaks

- Unit root with Fourier Change:

  - Fourier Dickey-Fuller Unit Root Test. (Enders and Lee, 2012)

$$Y_t = \alpha + \gamma t + \beta_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_2 \cos\left(\frac{2\pi kt}{T}\right) + \emptyset Y_{t-1} + \varepsilon_t,$$

$H_0 : \emptyset = 1$ v.s.

$H_1 : \emptyset < 1.$ (3)

Our Paper's Concern

- For Ø = 1 in (3), the size performance of traditional DF test.

  That is, If the true DGP is

$$Y_t = \alpha + \gamma t + \beta_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_2 \cos\left(\frac{2\pi kt}{T}\right) + \emptyset Y_{t-1} + \varepsilon_t,\ \emptyset = 1,$$

But $\beta_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_2 \cos\left(\frac{2\pi kt}{T}\right)$ is ignored.

What would happen to traditional DF test ?

Our Paper's Concern

- Main Results:

  - Mean Case (critical value is -2.88) :

$$t^{DF_{C,B}} \to 0.$$

Because -2.88 < 0, Null Hypothesis $\emptyset = 1$ is always accepted.

Our Paper's Concern

- Main Results:

  - Trend Case (critical value is -3.96) :

$$t^{DF_{t},B} \rightarrow \frac{\frac{6\varsigma_1\varsigma_2}{\pi k\sigma}}{\left(\frac{1}{2}(\varsigma_1^2+\varsigma_2^2)-\frac{3\varsigma_1^2}{(\pi k)^2}\right)^{1/2}}, \quad \text{(Trend)}$$

$\varsigma_1 * \varsigma_2 > 0$, Null Hypothesis $\emptyset = 1$ is **always accepted**.

$\varsigma_1 * \varsigma_2 < 0$ and kσ is small, then $t^{DF_{t},B}$ is a large negative number, therefore

Null Hypothesis $\emptyset = 1$ is **possible incorrectly rejected**.

Our Paper's Concern

- Main Results:
  - Figure

Figure (1a). kappa 1=2 and kappa 2=−2

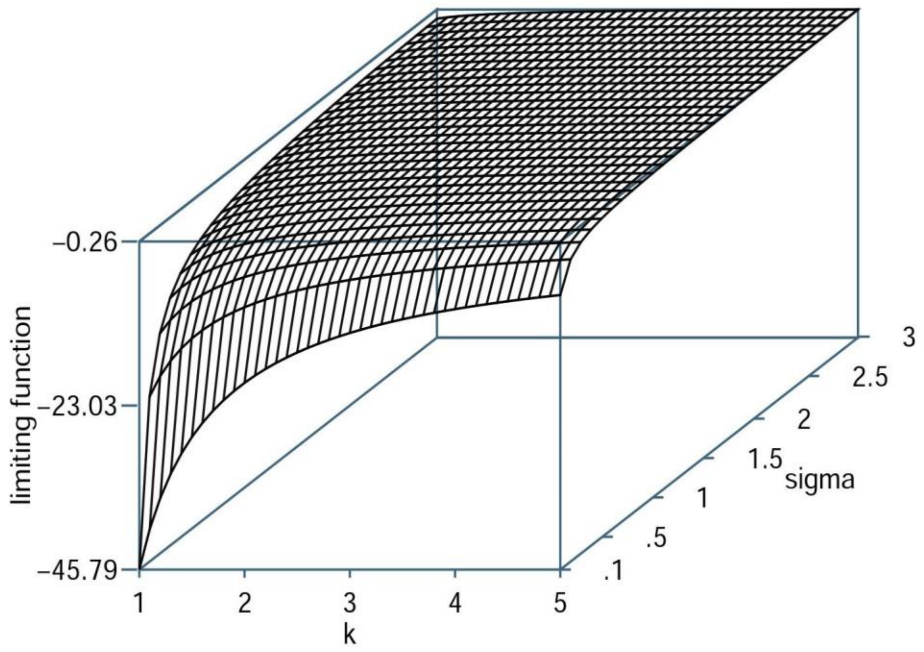
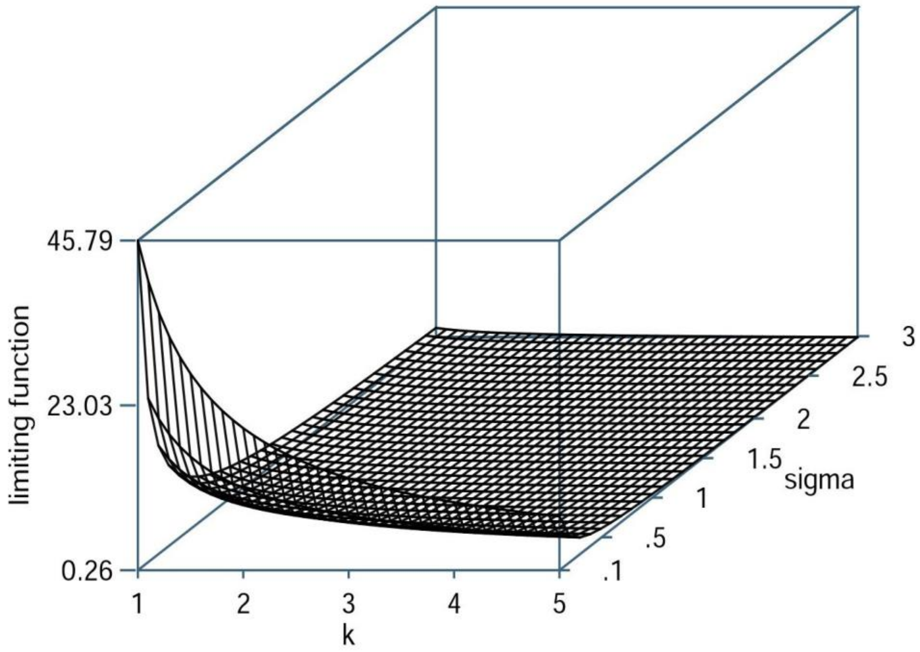


Figure (1b). kappa 1=2 and kappa 2=2



Our Paper's Concern

- Main Results:
  - Simulation Confirmations
    - Trend Case

Table 1

Size of standard DF test with trend when DGP is given as (1) and (2). (Under a fixed $\sigma = 1$)

T	(κ_1, κ_2)	(0.1,-0.1)	(0.1,0.1)	(0.5,-0.5)	(0.5,0.5)	(1,-1)	(1,1)	(1.5,-1.5)	(1.5,1.5)	(2,-2)	(2,2)
100	$k=1$	0.043	0.046	0.024	0.001	0.118	0.000	0.439	0.000	0.802	0.000
	$k=2$	0.048	0.055	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=3$	0.052	0.056	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=4$	0.045	0.043	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=5$	0.041	0.043	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
200	$k=1$	0.052	0.049	0.022	0.001	0.142	0.000	0.577	0.000	0.936	0.000
	$k=2$	0.055	0.054	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=3$	0.050	0.055	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=4$	0.046	0.047	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=5$	0.042	0.043	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
500	$k=1$	0.052	0.047	0.013	0.003	0.151	0.000	0.673	0.000	0.979	0.000
	$k=2$	0.051	0.052	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=3$	0.049	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=4$	0.045	0.048	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=5$	0.042	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: y_t is generated as $(1-\phi L)(y_t - \alpha(t) - \gamma t) = u_t$, $\alpha(t) = \alpha_0 + \beta_1 \sin(2\pi kt/T) + \beta_2 \cos(2\pi kt/T)$, with $\phi = 1$, $\beta_1 = \kappa_1\sqrt{T}$, $\beta_2 = \kappa_2\sqrt{T}$, $\gamma = 1$, $\alpha_0 = 1$ and $u_t \overset{i.i.d.}{\sim} N(0, \sigma^2)$ with $\sigma = 1$. Size is computed at the 5% nominal level, based on 2000 replications.

Our Paper's Concern

- Main Results:
  - Simulation Confirmations
    - Mean Case

Table 2

Size of standard trend-free DF test when DGP is given as (1) and (2) with $\gamma = 0$. (Under a fixed $\sigma = 1$)

T	(κ_1, κ_2)	(0.1,-0.1)	(0.1,0.1)	(0.5,-0.5)	(0.5,0.5)	(1,-1)	(1,1)	(1.5,-1.5)	(1.5,1.5)	(2,-2)	(2,2)
100	$k=1$	0.053	0.043	0.028	0.003	0.022	0.000	0.001	0.000	0.001	0.000
	$k=2$	0.048	0.037	0.011	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	$k=3$	0.039	0.040	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=4$	0.032	0.038	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=5$	0.031	0.036	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
200	$k=1$	0.049	0.048	0.034	0.002	0.023	0.000	0.008	0.000	0.003	0.000
	$k=2$	0.041	0.031	0.009	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	$k=3$	0.038	0.029	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=4$	0.043	0.034	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$k=5$	0.038	0.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
500	$k=1$	0.052	0.043	0.049	0.002	0.021	0.000	0.018	0.001	0.008	0.001
	$k=2$	0.045	0.028	0.017	0.001	0.007	0.000	0.002	0.000	0.001	0.000
	$k=3$	0.048	0.030	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	$k=4$	0.032	0.028	0.005	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	$k=5$	0.033	0.032	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: y_t is generated as in Table 1 with $\gamma = 0$. Size is computed at the 5% nominal level, based on 2000 replications.

Our Paper's Concern

- Main Results:
  - Simulation Confirmations
    - Trend Case

Table 3

Size of standard DF test with trend when DGP is given as (1) and (2). (Under fixed large κ's, $\kappa_1 = 1, \kappa_2 = -1$)

T	(κ_1, κ_2)=(1,-1)	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 2$	$\sigma = 2.5$
100	k =1	0.813	0.118	0.035	0.030	0.028
	k =2	0.000	0.000	0.000	0.000	0.001
	k =3	0.000	0.000	0.000	0.000	0.000
	k =4	0.000	0.000	0.000	0.000	0.000
	k =5	0.000	0.000	0.000	0.000	0.000
200	k =1	0.936	0.142	0.031	0.019	0.022
	k =2	0.000	0.000	0.000	0.000	0.000
	k =3	0.000	0.000	0.000	0.000	0.000
	k =4	0.000	0.000	0.000	0.000	0.000
	k =5	0.000	0.000	0.000	0.000	0.000
500	k =1	0.981	0.151	0.031	0.022	0.021
	k =2	0.000	0.000	0.000	0.000	0.000
	k =3	0.000	0.000	0.000	0.000	0.000
	k =4	0.000	0.000	0.000	0.000	0.000
	k =5	0.000	0.000	0.000	0.000	0.000

Note: y_t is generated as in Table 1 with various σ. Size is computed at the 5% nominal level, based on 2000 replications.

Our Paper's Concern

- Main Results:
  - Figure

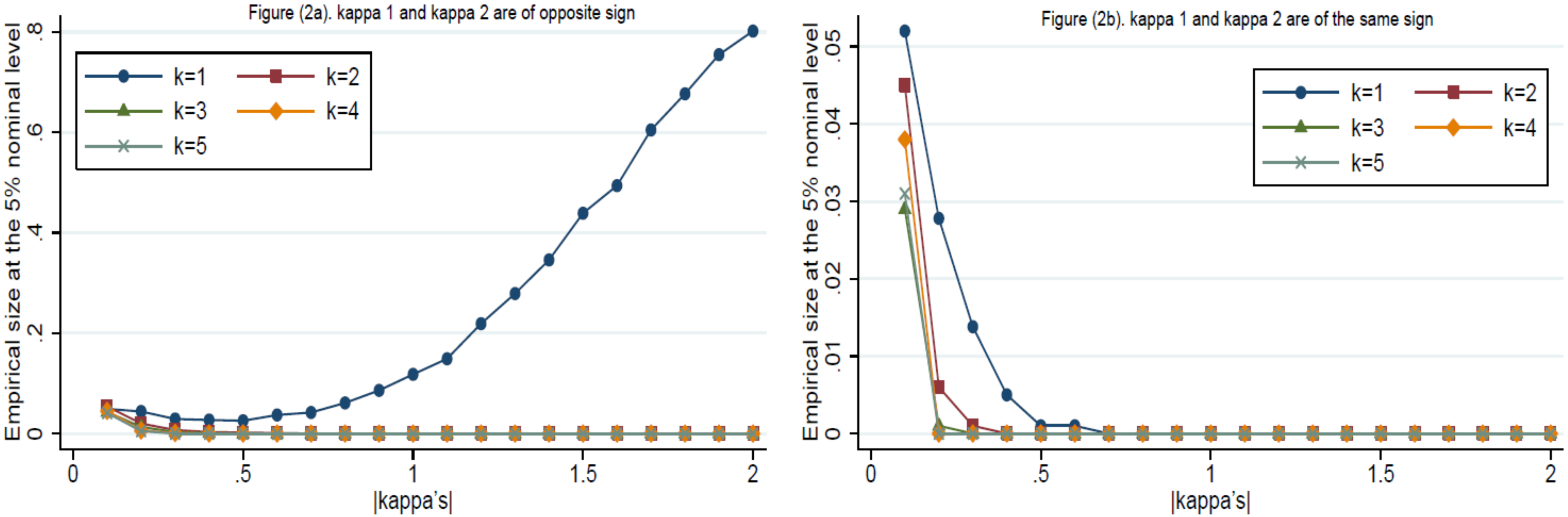


Figure 2. Size of standard DF test with trend when DGP is given as (1) and (2) with $\phi = 1$ and $u_t \sim N(0,1)$, $T = 100$.

Our Paper's Concern

- Main Results:
  - Figure

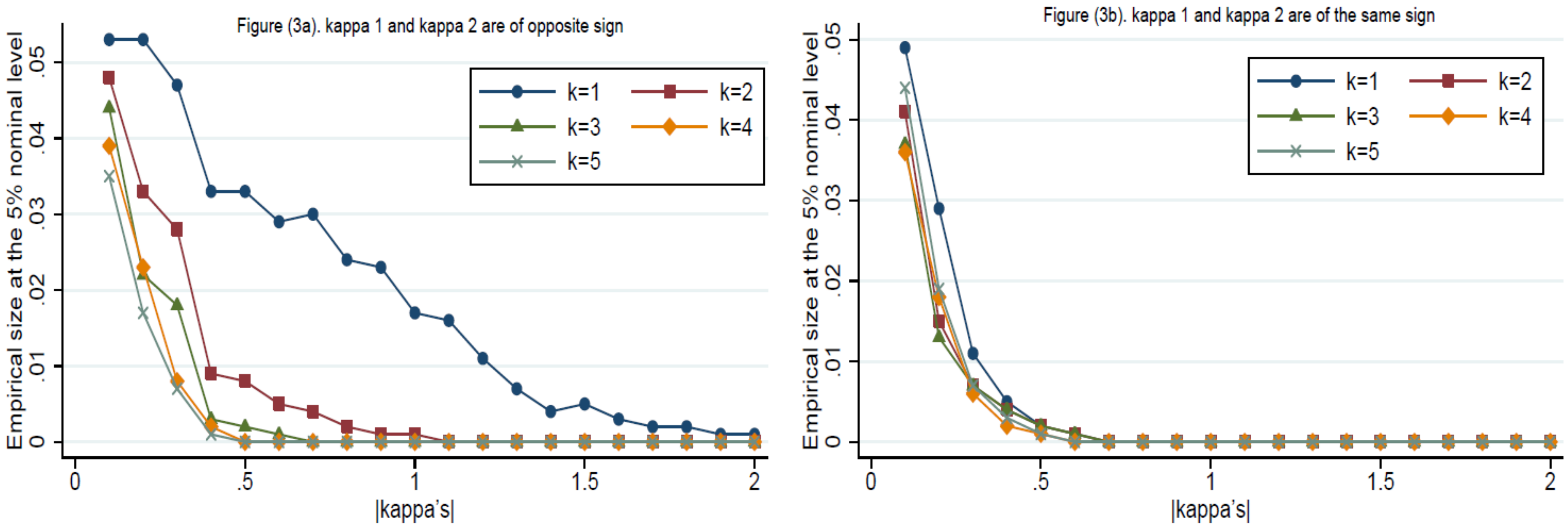


Figure 3. Size of standard trend-free DF test when DGP is given as (1) and (2) with $\gamma = 0$, $\phi = 1$ and $u_t \sim N(0,1)$, $T = 100$.

Conclusion

- The converse Perron phenomenon would possibly occur when there is a Fourier- form break.

  - Assume the null unit-root process is accompanied with a Fourier component.
    We derive the asymptotic distribution of the DF t-statistic. We find

    - Ignore the Fourier component will result in non-trivial size distortion

    - The null hypothesis of unit root can be either over- or under-rejected.

    - The converse Perron phenomenon is likely to occur if a low-frequency Fourier component is ignored and the underlying time series is containing a linear trend with the variance of regression errors is small.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