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A Simple Panel Unit-Root Test with Smooth Breaks in the Presence of a Multifactor Error Structure*

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Abstract

This paper extends the cross-sectionally augmented panel unit-root test (*CIPS*) developed by Pesaran *et al.* (2013, *Journal of Econometrics*, Vol. 175, pp. 94–115) to allow for smoothing structural changes in deterministic terms modelled by a Fourier function. The proposed statistic is called the break augmented *CIPS* (*BCIPS*) statistic. We show that the non-standard limiting distribution of the (truncated) *BCIPS* statistic exists and tabulate its critical values. Monte-Carlo experiments point out that the sizes and powers of the *BCIPS* statistic are generally satisfactory as long as the number of time periods, T, is not less than fifty. The *BCIPS* test is then applied to examine the validity of long-run purchasing power parity.

I. Introduction

The development of panel unit-root tests has been a hot research topic during the past decade. The first generation articles assume that idiosyncratic errors are cross-sectionally independent (Banerjee, 1999; Levin, Lin and Chu, 2002; Im, Pesaran and Shin, 2003, IPS; Maddala and Wu, 1999) and the second generation articles focus on the tests that allow cross-dependent errors (Chang, 2002; Breitung and Das, 2003; Phillips and Sul, 2003; Bai and Ng, 2004; Moon and Perron, 2004; Smith *et al.*, 2004; Choi and Chue, 2007; Pesaran, 2007; Pesaran, Smith and Yamagata, 2009, 2012, 2013). Nonetheless, these articles assume no structural changes in the models.

Two recent papers proposed panel unit-root tests that allow for multiple structural changes and cross-sectional dependence. Bai and Carrion-i-Silvestre (2009) propose a modified Sargan–Bhargava (1983, MSB) test in the panel setting. Although this test is invariant to both mean and trend break parameters, the limiting distribution of the indi-

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vidual MSB ($MSB^*(\lambda)$) test depends on the number of structural breaks. Following the cross-sectionally augmented procedure of Pesaran (2007), Im, Lee and Tieslau (2010, ILT) develop an LM-type panel unit-root test to account for possible heterogeneity in both the level and the trend of the series. The ILT test is invariant to nuisance parameters, but its limiting distribution depends on the number of trend breaks.

Instead of adopting dummy variables to capture discrete breaks, several articles develop unit-root tests by applying Gallant's (1981) flexible Fourier form to take into account smoothing breaks in the deterministic components (Becker, Enders and Hurn, 2004; Becker, Enders and Lee, 2006; Enders and Lee, 2012a,b; Rodrigues and Taylor, 2012). Enders and Lee (2012a,b) point out several advantages of the Fourier form approximation. First, it works reasonably well for types of breaks often used in economic analysis. Second, the Fourier function with a single-frequency component (κ) can be a reasonable approximation for breaks of an unknown form even if the function itself is not periodic. Third, it involves only the determination of the appropriate component in the model and hence avoids the complication of selecting break dates, the number of breaks and the form of breaks. Enders and Lee (2012a,b) find that their proposed tests are robust to a variety of possible break mechanisms in the deterministic trend function of unknown forms and numbers. Their Fourier tests complement the unit-root tests using dummy variables.

This paper extends Pesaran *et al.*'s (2013) multifactor error structure model to allow for smoothing breaks in deterministic components and then develops a new simple panel unit-root test that accommodates cross-sectional dependence among variables and smoothing changes in deterministic components. We first develop the breaks and cross-sectional dependence augmented *ADF* (*BCADF*) statistic and its average statistic by generalizing their cross-sectionally augmented *ADF* (*CADF*) regression to incorporate a single-frequency Fourier function with heterogeneous amplitudes. The breaks and cross-sectional dependence augmented *IPS* (*BCIPS*) statistic is proposed by averaging the *BCADF* statistics across individuals. An important advantage of the tests is their simplicity in empirical applications.

To analyse the impact of Fourier terms in the *BCADF* regression in both finite and infinite T, new asymptotic results of the *BCADF* and *BCIPS* statistics are derived based on the sequential and joint limit approaches respectively. In the case of serially uncorrelated errors, Theorems 1 and 2 show that the asymptotic distribution of the *BCADF* statistic does not depend on nuisance parameters when the number of individuals, N, tends to infinity under a fixed T or when both N and T sequentially and jointly tend to infinity. Theorem 3 examines the limiting distribution of the *CADF* statistic provided by Pesaran *et al.* (2013) when Fourier form breaks exist in the data-generating process (DGP) but are ignored in the regression. We show that, because of the omitted-variable bias, the asymptotic distribution of the *CADF* statistic under a fixed T depends on nuisance parameters even when N tends to infinity, but the dependence vanishes when both N and T approach infinity. Besides, the limiting distribution of the (truncated) *BCIPS* statistic is shown to exist. Theorem 4 shows that the *BCADF* statistic, under first-order autocorrelated errors, has the same asymptotic distribution as one that is obtained based on serially uncorrelated errors when both N and T tend to infinity. Furthermore, this paper extends the discussion to the case with a general autoregressive and moving average, $ARMA(l,s)$, specification of errors. In such a case, we suggest augmenting the *BCADF* regression with the lag order p. Although the asymptotic distribution of the *BCIPS* statistic exists, it is not analytically tractable since the non-attenuation in the dependence across individual *BCADF* statistics invalidates the application of the standard central limit theorem. This paper, therefore, tabulates the critical values of the *BCIPS* statistic under different N, T, κ and p by stochastic simulations and then explores its finite sample properties via Monte-Carlo simulations. The simulation results support that the limiting distribution of our proposed statistic does not depend on nuisance parameters, that the sizes (powers) of the statistic are generally good as long as $T \geqslant 50$ ($T \geqslant 100$), and that the power of the test increases with the Fourier frequency. On the other hand, the *CIPS* statistic of Pesaran *et al.* (2013) may reveal serious size distortions when the magnitude of break amplitudes is medium or large even for $T = 200$. Finally, the *BCIPS* test is applied to investigate the long-run purchasing power parity (PPP) over the post-Bretton Woods period.

The remainder of the paper is organized as follows. Section II sets out the basic dynamic heterogeneous panel data model with smooth breaks. The cross-sectional dependence across individuals is modelled by unobservable stationary common factors, and the smooth breaks in deterministic terms are captured by a single frequency Fourier function. In section III, we derive the null distribution of the individual *BCADF* statistic with serially uncorrelated errors, discuss the *BCADF*-based panel unit-root test and extend our results to the case with serially correlated errors. We also examine the limiting distribution of Pesaran *et al.*'s (2013) *CADF* statistic when Fourier form breaks exist in the DGP but are ignored in the regression. Section IV examines the finite-sample properties of the proposed *BCIPS* test via Monte-Carlo simulations. Section V provides an empirical application. Finally, section VI concludes. The proofs of the theorems are reported in Appendix S1. The simulated critical values and the finite sample properties of the *BCIPS* test under a linear trend model are reported in Appendix S2. Both supplementary appendices are not included in the paper but they are available in the journal webpage. Throughout this paper, the Fourier frequencies considered are assumed to be integer values only, $\xrightarrow{N}$ denotes convergence as $N \to \infty$; $\xrightarrow{T}$ denotes convergence as $T \to \infty$; $(N, T)_{seq} \to \infty$ denotes sequential convergence as $N \to \infty$ (first) and then $T \to \infty$; $(N, T)_j \to \infty$ denotes joint convergence as N and $T \to \infty$; $[Tr]$ denotes the largest integer not exceeding Tr and $\|\boldsymbol{A}\|$ denotes $\mathrm{tr}(\boldsymbol{AA}')^{1/2}$.

II. Breaks and the cross dependence panel data model

Let y_{it} be an observation on the ith cross-sectional unit at time t and suppose that it is generated according to the following simple dynamic linear heterogeneous panel data model with an unknown time-dependent intercept term $\delta_i(t)$:

$$(1-\phi_i L)(y_{it}-\delta_i(t)-\varsigma_i t)=u_{it}, \quad u_{it}=\boldsymbol{\gamma}_{iy}'\boldsymbol{f}_t+\varepsilon_{iyt}, \quad t=1,..,T;\ i=1,..,N, \qquad (1)$$

where $\varsigma_i t$ is a linear trend, $\boldsymbol{f}_t$ is an $m \times 1$ unobserved stationary stochastic common factor, $\boldsymbol{\gamma}_{iy}$ is the associated factor loading reflecting the degree of contemporaneous correlation across individuals, and ε_{iyt} is an idiosyncratic error. We begin our analysis with a DGP containing only a single Fourier frequency (κ) since it mimics a variety of breaks in deterministic components (Enders and Lee, 2012a):[1]

$$\delta_i(t) = \varpi_{i,\kappa,t} = \mu_i + \alpha_{iy,1}\sin(2\pi\kappa t/T) + \alpha_{iy,2}\cos(2\pi\kappa t/T), \tag{2}$$

where κ is the frequency parameter reflecting the number of cycles in the sample period and is assumed to be homogeneous across agents, and $\alpha_{iy,1}$ and $\alpha_{iy,2}$ measure the heterogeneous amplitude and displacement of sinusoidal components across agents, respectively. $\varpi_{i,\kappa,t}$ in equation (2) captures smooth breaks in the intercept.[2] Assuming a homogeneous κ across individuals is not so restrictive since it does not necessarily imply an identical number of breaks across individuals. This is because variations in $\alpha_{iy,1}$ and $\alpha_{iy,2}$ accommodate, to some degree, different breaks for each individual. Substituting equation (2) into (1), we obtain:

$$\Delta y_{it} = \beta_i y_{i,t-1} - \beta_i \boldsymbol{\alpha}'_{iy}\boldsymbol{d}_t + \phi_i \boldsymbol{\alpha}'_{iy}\Delta\boldsymbol{d}_t + \boldsymbol{\gamma}'_{iy}\boldsymbol{f}_t + \varepsilon_{iyt}, \quad t = 1,..,T;\ i = 1,..,N, \tag{3}$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$, $\beta_i = \phi_i - 1$, $\boldsymbol{d}_t = (1, \sin(2\pi\kappa t/T), \cos(2\pi\kappa t/T), t)'$ is a 4×1 vector of deterministic common components, $\Delta\boldsymbol{d}_t = (0, \Delta\sin(2\pi\kappa t/T), \Delta\cos(2\pi\kappa t/T), 1)'$ and $\boldsymbol{\alpha}_{iy} = (\mu_i, \alpha_{iy,1}, \alpha_{iy,2}, \varsigma_i)'$. Without loss of generality, it is assumed that $\boldsymbol{d}_0 = \mathbf{0}$. The unit-root hypothesis, $\phi_i = 1$ for all i, can be expressed as:

$$H_0 : \beta_i = 0, \quad \forall i, \tag{4}$$

against the possibly heterogeneous alternative,

$$H_1 : \beta_i < 0, \quad i = 1, 2, \ldots, N_1;\ \beta_i = 0, i = N_1 + 1, N_1 + 2, \ldots, N. \tag{5}$$

Under the above null hypothesis that $\beta_i = 0$ ($\phi_i = 1$), equation (3) becomes

$$\Delta y_{it} = \boldsymbol{\alpha}'_{iy}\Delta\boldsymbol{d}_t + \boldsymbol{\gamma}'_{iy}\boldsymbol{f}_t + \varepsilon_{iyt}, \quad t = 1,..,T;\ i = 1,..,N.$$

After recursively substituting $y_{i,t-j}, j = 1,\ldots, t-1$ in the above equation and assuming that $\boldsymbol{d}_0 = \mathbf{0}$, we can obtain the following equation for y_{it}:

$$y_{it} = y_{i0} + \boldsymbol{\alpha}'_{iy}\boldsymbol{d}_t + \boldsymbol{\gamma}'_{iy}\boldsymbol{s}_{ft} + s_{iyt}, \tag{6}$$

where $\boldsymbol{s}_{ft} = \boldsymbol{f}_1 + \boldsymbol{f}_2 + \cdots + \boldsymbol{f}_t$ and $s_{iyt} = \varepsilon_{iy1} + \varepsilon_{iy2} + \cdots + \varepsilon_{iyt}$. Therefore, under H_0, y_{it} is composed of a deterministic component with a Fourier element, $y_{i0} + \boldsymbol{\alpha}'_{iy}\boldsymbol{d}_t$; a common stochastic component, $\boldsymbol{s}_{ft} \sim I(1)$; and an idiosyncratic component, $s_{iyt} \sim I(1)$. We do not assume that $\alpha_{iy,1} = \alpha_{iy,2} = 0$ under the null hypothesis, and hence heterogeneous breaks exist under the null and alternative hypotheses of equations (4) and (5) respectively. Our proposed tests in the following section avoid the possibility of spuriously rejecting a unit-root hypothesis (Enders and Lee, 2009).

[1] Allowing for two frequency parameters, $\kappa_1 = 1$ and $\kappa_2 = 2$, is important if breaks are sharp (Enders and Lee, 2012a).

[2] Introducing a time trend, $\varsigma_i t$, in equation (1) removes the restriction that the starting and ending values of the Fourier function are the same. Changes in the intercept and slope of a deterministic function can be captured by the Fourier approximation. Hence, our proposed panel unit-root tests allow for breaks in both the level and trend of the series under investigation.

III. Breaks and cross dependence augmented unit-root tests

Theorems 1–4 in this section derive the asymptotic distribution of the unit-root test statistic under the null hypothesis in equation (4) for the ith individual. Note that all of the order results and proofs of theorems given in Appendix S1 are derived from the case where $\boldsymbol{d}_t = (1, \sin(2\pi\kappa t/T), \cos(2\pi\kappa t/T))'$, $t = 1, 2, \ldots, T$. The asymptotic results for the case where $\boldsymbol{d}_t = (1, \sin(2\pi\kappa t/T), \cos(2\pi\kappa t/T), t)'$ can be derived in a similar manner.

Unit-root tests in the presence of multiple factors

In the case where m unobservable factors ($m > 1$) exist, we need at least m equations to solve for them. Following Pesaran *et al.* (2013), we assume that, in addition to y_{it}, there exist $k\,(k + 1 \geqslant m)$ additional variables, $\boldsymbol{x}_{it}, i = 1, \ldots, k$, depending on at least the same set of common factors, $\boldsymbol{s}_{ft}$. Suppose that the $k \times 1$ vector of additional variables follows the general linear process:

$$\Delta \boldsymbol{x}_{it} = \boldsymbol{A}_{ix}\Delta \boldsymbol{d}_t + \boldsymbol{\Gamma}_{ix}\boldsymbol{f}_t + \boldsymbol{\varepsilon}_{ixt}, \quad i = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T, \tag{7}$$

where $\boldsymbol{x}_{it} = (x_{i1t}, x_{i2t}, \ldots, x_{ikt})'$, $\boldsymbol{A}_{ix} = (\boldsymbol{a}_{ix1}, \boldsymbol{a}_{ix2}, \ldots, \boldsymbol{a}_{ixk})'$, $\boldsymbol{\Gamma}_{ix} = (\gamma_{ix1}, \gamma_{ix2}, \ldots, \gamma_{ixk})'$, and $\boldsymbol{\varepsilon}_{ixt}$ is the idiosyncratic component of $\boldsymbol{x}_{it}$ and is distributed independently of ε_{iys} for all i, t and s. The level equation of $\boldsymbol{x}_{it}$ can be obtained by recursively substituting equation (7):

$$\boldsymbol{x}_{it} = \boldsymbol{x}_{i0} + \boldsymbol{A}_{ix}\boldsymbol{d}_t + \boldsymbol{\Gamma}_{ix}\boldsymbol{s}_{ft} + \boldsymbol{s}_{ixt}, \quad i = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T, \tag{8}$$

where $\boldsymbol{s}_{ixt} = \sum_{s=1}^{t} \boldsymbol{\varepsilon}_{ixs}$. Combining equations (6) and (8), we have the null data generating process:

$$\boldsymbol{z}_{it} = \boldsymbol{z}_{i0} + \boldsymbol{A}_i\boldsymbol{d}_t + \boldsymbol{\Gamma}_i\boldsymbol{s}_{ft} + \boldsymbol{s}_{it}, \tag{9}$$

where $\boldsymbol{z}_{it} = (y_{it}, \boldsymbol{x}_{it}')'$, $\boldsymbol{A}_i = (\boldsymbol{\alpha}_{iy}, \boldsymbol{A}_{ix}')' \equiv (\boldsymbol{\mu}_i, \boldsymbol{\alpha}_{i,1}, \boldsymbol{\alpha}_{i,2}, \varsigma_i)$, $\boldsymbol{\Gamma}_i = (\gamma_{iy}, \boldsymbol{\Gamma}_{ix}')'$, and $\boldsymbol{s}_{it} = (s_{iyt}, \boldsymbol{s}_{ixt}')'$. An assumption for the initial condition $\boldsymbol{z}_{i0}$ is given in Assumption 4 appearing before equation (16).

To obtain observable proxies for the unobserved common effect $\boldsymbol{f}_t$, we first combine equations (3) and (7) and present the resulting equations in matrix form. The difference equation (not necessary under the null hypothesis) of $\boldsymbol{z}_i$ is:

$$\Delta \boldsymbol{z}_i = \boldsymbol{z}_{i,-1}\boldsymbol{B}_i' + \boldsymbol{D}\boldsymbol{C}_i' + \boldsymbol{F}\boldsymbol{\Gamma}_i' + \Delta \boldsymbol{D}\ddot{\boldsymbol{A}}_i' + \boldsymbol{\varepsilon}_i, \tag{10}$$

where $\Delta \boldsymbol{z}_i = (\Delta \boldsymbol{z}_{i1}, \Delta \boldsymbol{z}_{i2}, \ldots, \Delta \boldsymbol{z}_{iT})'$, $\boldsymbol{z}_{i,-1} = (\boldsymbol{z}_{i0}, \boldsymbol{z}_{i1}, \ldots, \boldsymbol{z}_{iT-1})'$, $\boldsymbol{B}_i = (\beta_i, 0')'$, $\boldsymbol{D} = (\boldsymbol{d}_1, \boldsymbol{d}_2, \ldots, \boldsymbol{d}_T)'$, $\boldsymbol{C}_i = (-\beta_i\boldsymbol{\alpha}_{iy}', 0')'$, $\boldsymbol{F} = (\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_T)'$, $\Delta \boldsymbol{D} = (\Delta \boldsymbol{d}_1, \Delta \boldsymbol{d}_2, \ldots, \Delta \boldsymbol{d}_T)'$, $\ddot{\boldsymbol{A}}_i = (\phi_i\boldsymbol{\alpha}_{iy}, \boldsymbol{A}_{ix}')'$ and $\boldsymbol{\varepsilon}_i = (\boldsymbol{\varepsilon}_{i1}, \boldsymbol{\varepsilon}_{i2}, \ldots, \boldsymbol{\varepsilon}_{iT})'$ with $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{iyt}, \boldsymbol{\varepsilon}_{ixt}')'$. Taking the cross-sectional average of equation (10), we obtain:

$$\Delta \bar{\boldsymbol{z}} = \overline{\boldsymbol{z}_{-1}\boldsymbol{B}} + \boldsymbol{D}\overline{\boldsymbol{C}} + \boldsymbol{F}\overline{\boldsymbol{\Gamma}}' + \Delta \boldsymbol{D}\overline{\ddot{\boldsymbol{A}}}' + \bar{\boldsymbol{\varepsilon}}, \tag{11}$$

where $\Delta \bar{\boldsymbol{z}} = N^{-1}\sum_{i=1}^{N} \Delta \boldsymbol{z}_i$, $\overline{\boldsymbol{z}_{-1}\boldsymbol{B}} = N^{-1}\sum_{i=1}^{N} \boldsymbol{z}_{i,-1}\boldsymbol{B}_i$, $\overline{\boldsymbol{C}} = N^{-1}\sum_{i=1}^{N} \boldsymbol{C}_i$, $\overline{\boldsymbol{\Gamma}} = N^{-1}\sum_{i=1}^{N} \boldsymbol{\Gamma}_i$, $\overline{\ddot{\boldsymbol{A}}} = N^{-1}\sum_{i=1}^{N} \ddot{\boldsymbol{A}}_i$ and $\bar{\boldsymbol{\varepsilon}} = N^{-1}\sum_{i=1}^{N} \boldsymbol{\varepsilon}_i$. If $\overline{\boldsymbol{\Gamma}}$ has full rank then $\boldsymbol{F}$ in equation (11) can be solved as:

$$\boldsymbol{F} = (\Delta \bar{\boldsymbol{z}} - \overline{\boldsymbol{z}_{-1}\boldsymbol{B}} - \boldsymbol{D}\overline{\boldsymbol{C}} - \Delta \boldsymbol{D}\overline{\ddot{\boldsymbol{A}}}' - \bar{\boldsymbol{\varepsilon}})\overline{\boldsymbol{\Gamma}}(\overline{\boldsymbol{\Gamma}}'\overline{\boldsymbol{\Gamma}})^{-1}. \tag{12}$$

Pesaran *et al.* (2013) showed that $\bar{\varepsilon} \xrightarrow{N} 0$ for each t. Hence, we obtain:

$$\boldsymbol{F} - (\Delta\bar{z} - \overline{z_{-1}\boldsymbol{B}} - \boldsymbol{D}\overline{\boldsymbol{C}} - \Delta\boldsymbol{D}\overline{\boldsymbol{A}}')\overline{\boldsymbol{\Gamma}}(\overline{\boldsymbol{\Gamma}}'\overline{\boldsymbol{\Gamma}})^{-1} \xrightarrow{N} 0. \tag{13}$$

The linear combination of $(\bar{z}_{-1}, \Delta\bar{z}, \boldsymbol{D}, \Delta\boldsymbol{D})$ in equation (13) is a reasonable proxy for $\boldsymbol{f}_t$. After substituting $\boldsymbol{f}_t$ in equation (3) by $\bar{z}_{-1}$, $\Delta\bar{z}$, $\boldsymbol{D}$ and $\Delta\boldsymbol{D}$, and using the results of (L5) and (L6) in Appendix S2, we suggest regressing the following breaks and cross dependence augmented Dickey–Fuller equation for each individual by OLS:[3]

$$\begin{aligned}\Delta y_{it} = c_{i,0} + c_{i,1}\sin(2\pi\kappa t/T) + c_{i,2}\cos(2\pi\kappa t/T) + \boldsymbol{c}_{i,3}'\bar{\boldsymbol{z}}_{t-1} \\ + \boldsymbol{c}_{i,4}'\Delta\bar{\boldsymbol{z}}_t + b_i y_{i,t-1} + e_{it}, t = 1, 2, \ldots, T.\end{aligned} \tag{14}$$

The t-statistic of the estimate of b_i ($\hat{b}_i$) is applied to examine the unit-root hypothesis and is expressed as:

$$t_i(N,T) = \frac{\Delta\boldsymbol{y}_i'\boldsymbol{M}_z\boldsymbol{y}_{i,-1}}{\hat{\sigma}_i(\boldsymbol{y}_{i,-1}'\boldsymbol{M}_z\boldsymbol{y}_{i,-1})^{1/2}}, \tag{15}$$

where $\Delta\boldsymbol{y}_i = (\Delta y_{i1}, \Delta y_{i2}, \ldots, \Delta y_{iT})'$, $\boldsymbol{y}_{i,-1} = (y_{i0}, y_{i1}, \ldots, y_{i,T-1})'$, $\boldsymbol{M}_z = \boldsymbol{I}_T - \boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'$, $\boldsymbol{Z} = (\Delta\bar{z}, \boldsymbol{\tau}, \boldsymbol{\Upsilon}_1, \boldsymbol{\Upsilon}_2, \bar{z}_{-1})$, $\boldsymbol{\tau} = (1, 1, \ldots, 1)'$, $\boldsymbol{\Upsilon}_1 = (\sin(2\pi\kappa 1/T), \sin(2\pi\kappa 2/T), \ldots, \sin(2\pi\kappa T/T))'$, $\boldsymbol{\Upsilon}_2 = (\cos(2\pi\kappa 1/T), \cos(2\pi\kappa 2/T), \ldots, \cos(2\pi\kappa T/T))'$, $\Delta\bar{z} = (\Delta\bar{z}_1, \Delta\bar{z}_2, \ldots, \Delta\bar{z}_T)'$, $\bar{z}_{-1} = (\bar{z}_0, \bar{z}_1, \ldots, \bar{z}_{T-1})'$, and $\hat{\sigma}_i^2 = \frac{\Delta\boldsymbol{y}_i'\boldsymbol{M}_{i,z}\Delta\boldsymbol{y}_i}{T-2k-6}$, in which $\boldsymbol{M}_{i,z} = \boldsymbol{I}_T - \boldsymbol{G}_i(\boldsymbol{G}_i'\boldsymbol{G}_i)^{-1}\boldsymbol{G}_i'$ and $\boldsymbol{G}_i = (\boldsymbol{Z}, \boldsymbol{y}_{i,-1})$.

Following Pesaran *et al.* (2013), the required assumptions for deriving the null distribution of the $t_i(N,T)$ statistic are given as follows:

Assumption 1 (Idiosyncratic errors). The idiosyncratic error, ε_{iyt}, with a zero mean, a constant variance σ_i^2, $(0 < \sigma_i^2 \leqslant K)$ and a finite fourth-order moment, is independently distributed across i and t and is independent of $\boldsymbol{f}_s$ for all i, t, s.

Assumption 2 (Common factors). The $m \times 1$ vector of common factors, $\boldsymbol{f}_t$, follows a covariance stationary process with absolute summable autocovariance and is distributed independently of ε_{iys} for all i, t and s. Specifically, we assume that $\boldsymbol{f}_t = \boldsymbol{\Psi}(L)\boldsymbol{v}_t$, where $\boldsymbol{\Psi}^{-1}(1) \equiv \boldsymbol{\Lambda}_f^{-1}$ exists and $\boldsymbol{v}_t \sim \text{i.i.d.}(0, \boldsymbol{\Omega}_m)$ has a finite fourth-order moment.

Assumption 3 (Factor loadings). $\|\boldsymbol{A}_i\| \leqslant K$ and $\|\boldsymbol{\Gamma}_i\| \leqslant K$ for all i, with the factors normalized such that $E(\boldsymbol{f}_t\boldsymbol{f}_t') = \boldsymbol{I}_m$.

Assumption 4 (Initial conditions). $E\|\boldsymbol{s}_{f1}\| \leqslant K$, $E\|\boldsymbol{z}_{i0}\| \leqslant K$ and $E\|\boldsymbol{s}_{i1}\| \leqslant K$ for all i.

Assumption 5 (Rank condition). The $(k+1) \times m$ matrix of factor loadings, $\boldsymbol{\Gamma}_i$, satisfies the following condition:

$$\begin{aligned}\text{rank}(\overline{\boldsymbol{\Gamma}}) = m \leqslant k+1, \text{ for any } N \text{ and} \\ \overline{\boldsymbol{\Gamma}} \xrightarrow{N} \boldsymbol{\Gamma}^*,\end{aligned} \tag{16}$$

where $\boldsymbol{\Gamma}^*$ is a fixed bounded matrix with rank m.

[3]The terms in $\Delta\boldsymbol{D}$ can be ignored since $\Delta\sin(2\pi\kappa t/T) = 2\pi\kappa/T\cos(2\pi\kappa t/T) + o(1)$ and $\Delta\cos(2\pi\kappa t/T) = -2\pi\kappa/T\sin(2\pi\kappa t/T) + o(1)$. We appreciate a reviewer's comment.

Assumption 6 (Fourier amplitude coefficients). The Fourier amplitude coefficients $\boldsymbol{\alpha}_{i,1}$ and $\boldsymbol{\alpha}_{i,2}$ are non-random parameters.

For fixed N and T, the distribution of $t_i(N,T)$ depends on nuisance parameters through their effects on the matrices $\boldsymbol{M}_z$ and $\boldsymbol{M}_{i,z}$. However, Theorems 1 and 2 below show that this dependence vanishes either as $N \to \infty$, for a fixed T, or as N and $T \to \infty$, jointly. In the case of a fixed T, however, the effect of the initial cross-sectional mean $\bar{z}_0$ must be eliminated in order to ensure that $t_i(N,T)$ does not depend on nuisance parameters.[4] This can be achieved by working with the deviation from $\bar{z}_0$, $z_{it} - \bar{z}_0$.

Theorem 1. Let z_{it} be generated based on equation (9) with the cross-sectional mean of the initial observation $\bar{z}_0$ being zero. Suppose that Assumptions 1–6 hold. Then, the limiting distribution of $t_i(N,T)$ given by (15) will be free of nuisance parameters as $N \to \infty$ for any fixed $T > 2k + 6$. In particular, we have

$$t_i(N,T) \xrightarrow{N} \frac{\frac{\varepsilon'_{iy}\boldsymbol{s}_{iy,-1}}{\sigma_i^2 T} - q'_{iT}\boldsymbol{\Psi}_{fT}^{-1}\boldsymbol{h}_{iT}}{\left(\frac{\varepsilon'_{iy}\varepsilon_{iy}}{\sigma_i^2(T-2k-6)} - \frac{d'_{iT}\Xi_{iT}^{-1}d_{iT}}{T-2k-6}\right)^{1/2} \times \left(\frac{s'_{iy,-1}\boldsymbol{s}_{iy,-1}}{\sigma_i^2 T^2} - \boldsymbol{h}'_{iT}\boldsymbol{\Psi}_{fT}^{-1}\boldsymbol{h}_{iT}\right)^{1/2}}, \tag{17}$$

where $\boldsymbol{\varepsilon}'_{iy} = (\varepsilon_{iy1}, \varepsilon_{iy2}, \ldots, \varepsilon_{iyT})$, $\boldsymbol{s}'_{iy,-1} = (0, s_{iy,1}, \ldots, s_{iy,T-1})$,

$$\boldsymbol{q}_{iT} = \left[\frac{\boldsymbol{\varepsilon}'_{iy}\boldsymbol{F}}{\sigma_i\sqrt{T}} \quad \frac{\boldsymbol{\varepsilon}'_{iy}\boldsymbol{\tau}}{\sigma_i\sqrt{T}} \quad \frac{\boldsymbol{\varepsilon}'_{iy}\boldsymbol{\Upsilon}_1}{\sigma_i\sqrt{T}} \quad \frac{\boldsymbol{\varepsilon}'_{iy}\boldsymbol{\Upsilon}_2}{\sigma_i\sqrt{T}} \quad \frac{\boldsymbol{\varepsilon}'_{iy}\boldsymbol{s}_{f,-1}}{\sigma_i T}\right]', \boldsymbol{d}_{iT} \equiv \left[\boldsymbol{q}'_{iT} \quad \frac{\boldsymbol{s}'_{iy,-1}\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2 T}\right]', \tag{18}$$

$$\boldsymbol{h}_{iT} = \left[\frac{\boldsymbol{s}'_{iy,-1}\boldsymbol{F}}{\sigma_i T^{3/2}} \quad \frac{\boldsymbol{s}'_{iy,-1}\tau}{\sigma_i T^{3/2}} \quad \frac{\boldsymbol{s}'_{iy,-1}\boldsymbol{\Upsilon}_1}{\sigma_i T^{3/2}} \quad \frac{\boldsymbol{s}'_{iy,-1}\boldsymbol{\Upsilon}_2}{\sigma_i T^{3/2}} \quad \frac{\boldsymbol{s}'_{iy,-1}\boldsymbol{s}_{f,-1}}{\sigma_i T^2}\right]', \tag{19}$$

$$\boldsymbol{\Psi}_{fT} = \begin{bmatrix} \frac{\boldsymbol{F}'\boldsymbol{F}}{T} & \frac{\boldsymbol{F}'\tau}{T} & \frac{\boldsymbol{F}'\boldsymbol{\Upsilon}_1}{T} & \frac{\boldsymbol{F}'\boldsymbol{\Upsilon}_2}{T} & \frac{\boldsymbol{F}'\boldsymbol{s}_{f,-1}}{T^{3/2}} \\ \frac{\tau'\boldsymbol{F}}{T} & \frac{\tau'\tau}{T} & \frac{\tau'\boldsymbol{\Upsilon}_1}{T} & \frac{\tau'\boldsymbol{\Upsilon}_2}{T} & \frac{\tau'\boldsymbol{s}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\Upsilon}'_1\boldsymbol{F}}{T} & \frac{\boldsymbol{\Upsilon}'_1\tau}{T} & \frac{\boldsymbol{\Upsilon}'_1\boldsymbol{\Upsilon}_1}{T} & \frac{\boldsymbol{\Upsilon}'_1\boldsymbol{\Upsilon}_2}{T} & \frac{\boldsymbol{\Upsilon}'_1\boldsymbol{s}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\Upsilon}'_2\boldsymbol{F}}{T} & \frac{\boldsymbol{\Upsilon}'_2\tau}{T} & \frac{\boldsymbol{\Upsilon}'_2\boldsymbol{\Upsilon}_1}{T} & \frac{\boldsymbol{\Upsilon}'_2\boldsymbol{\Upsilon}_2}{T} & \frac{\boldsymbol{\Upsilon}'_2\boldsymbol{s}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{s}'_{f,-1}\boldsymbol{F}}{T^{3/2}} & \frac{\boldsymbol{s}'_{f,-1}\tau}{T^{3/2}} & \frac{\boldsymbol{s}'_{f,-1}\boldsymbol{\Upsilon}_1}{T^{3/2}} & \frac{\boldsymbol{s}'_{f,-1}\boldsymbol{\Upsilon}_2}{T^{3/2}} & \frac{\boldsymbol{s}'_{f,-1}\boldsymbol{s}_{f,-1}}{T^2} \end{bmatrix}, \text{ and} \tag{20}$$

$$\boldsymbol{\Xi}_{iT} = \begin{bmatrix} \boldsymbol{\Psi}_{fT} & \boldsymbol{h}_{iT} \\ \boldsymbol{h}'_{iT} & \frac{\boldsymbol{s}'_{iy,-1}\boldsymbol{s}_{iy,-1}}{\sigma_i^2 T^2} \end{bmatrix}. \tag{21}$$

Proof. See Appendix S1.

Theorem 2. Let z_{it} be generated based on equation (9) with the cross-sectional mean of the initial observation $\bar{z}_0$ being zero. Suppose that Assumptions 1–6 hold. Then, the limiting null distribution of $t_i(N,T)$ given by equation (15) will be free of nuisance parameters. In particular, the $t_i(N,T)$ statistic has the same sequential ($(N,T)_{seq} \to \infty$) and joint ($(N,T)_j \to \infty$) limiting distribution, referred to as the *BCADF* distribution,

[4] The importance of initial values to the power of the standardized, averaged Dickey–Fuller panel unit root statistic of IPS (2003) is discussed in Harris *et al.* (2010). A further investigation of this issue for a model with Fourier form breaks and cross-sectionally dependent errors is worthwhile, but it will not be pursued in this paper.

given by:

$$BCADF_{if} = \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{q}'_{if}\boldsymbol{\Psi}_f^{-1}\boldsymbol{h}_{if}}{\left(\int_0^1 W_i^2(r)d(r) - \boldsymbol{h}'_{if}\boldsymbol{\Psi}_f^{-1}\boldsymbol{h}_{if}\right)^{1/2}}, \tag{22}$$

where

$$\boldsymbol{q}_{if} = \begin{bmatrix} W_i(1) \\ -2\pi\kappa\int_0^1 \cos(2\pi\kappa r)W_i(r)dr \\ W(1) + 2\pi\kappa\int_0^1 \sin(2\pi\kappa r)W_i(r)dr \\ \int_0^1 [\boldsymbol{W_f}(r)]dW_i(r) \end{bmatrix}, \tag{23}$$

$$\boldsymbol{h}_{if} = \begin{bmatrix} \int_0^1 W_i(r)dr \\ -2\pi\kappa\left(\int_0^1 \cos(2\pi\kappa r)\left[\int_0^r W_i(s)ds\right]dr\right) \\ \int_0^1 W_i(s)ds + 2\pi\kappa\int_0^1 \sin(2\pi\kappa r)\left[\int_0^r W_i(s)ds\right]dr \\ \int_0^1 [\boldsymbol{W_f}(r)]W_i(r)dr \end{bmatrix}, \tag{24}$$

$$\boldsymbol{\Psi}_f = \begin{bmatrix} \boldsymbol{H}_{3\times 3} & \boldsymbol{R}_{3\times m} \\ \boldsymbol{R}'_{m\times 3} & \boldsymbol{J}_{m\times m} \end{bmatrix} \tag{25}$$

with

$$\boldsymbol{H}_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}, \boldsymbol{R}_{3\times m} = \begin{bmatrix} \int_0^1 [\boldsymbol{W_f}(r)]'dr \\ -2\pi\kappa\left(\int_0^1 \cos(2\pi\kappa r)\left[\int_0^r [\boldsymbol{W_f}(s)]'ds\right]dr\right) \\ \int_0^1 [\boldsymbol{W_f}(s)]'ds + 2\pi\kappa\int_0^1 \sin(2\pi\kappa r)\left[\int_0^r [\boldsymbol{W_f}(s)]'ds\right]dr \end{bmatrix},$$

and $\boldsymbol{J}_{m\times m} = \int_0^1 [\boldsymbol{W_f}(r)][\boldsymbol{W_f}(r)]'dr$. Here, $W_i(r)$ and $\boldsymbol{W_f}(r)$ are scalar and m-dimensional standard Brownian motions, respectively. $W_i(r)$ and $\boldsymbol{W_f}(r)$ are mutually independent. For the joint-limiting distribution to hold, it is also required that $N/T \to l$ as $(N,T)_j \to \infty$, where l is a non-zero finite positive constant.

Proof. See Appendix S1.

Theorem 2 shows that the asymptotic distribution of $t_i(N,T)$ depends only on the frequency parameter, κ, but is invariant to all other parameters in the DGP (equation (9)). Hence, the $t_i(N,T)$ statistic is a pivotal statistic. It is worth noting that the $t_i(N,T)$s, for $i = 1,\ldots, N$, are dependently distributed with the same degree of dependence since $BCADF_{if}$ and $BCADF_{jf}$, $\forall i \neq j$ $(i,j \in N)$, are nonlinear functions of the common process $\boldsymbol{W_f}(r)$, as can be seen from equations (22)–(25). Therefore, the standard central limit theorem cannot be applied to construct the standardized panel statistic based on the cross-sectional average of $t_i(N,T)s$ because of the non-attenuation in the dependence across $t_i(N,T)s$.

Remark 1. If we assume a specific frequency, κ_i, for individual i such that $\kappa_i \neq \kappa_j$, $\forall i \neq j$, $i,j \in N$, the individual unit root test statistic is denoted as $t_i(N,T,\kappa_i)$. The limiting distribution of $t_i(N,T,\kappa_i)$ depends on the frequency component κ_i, and its proof is sketched in Appendix S1. Because the limiting distributions of $t_i(N,T,\kappa_i)s$, $\forall i$, depend on the common process $\boldsymbol{W_f}(r)$, they are not cross-sectionally independent. Hence, the distribution of the standardized panel statistics is non-standard even for sufficiently large N. Since the critical values of our proposed panel unit root test can only be constructed by stochastic simulation, it is not empirically feasible to simulate critical values for all possible combinations of κ_i $(i = 1,\ldots, N)$ across individuals.

Pesaran *et al.*'s (2013) *CADF* Statistic under Fourier Form Breaks

Leybourne, Mills and Newbold (1998) and Leybourne and Newbold (2000) show that the standard Dickey–Fuller tests lead to a spurious rejection of the unit root hypothesis if a single instantaneous break occurs in the beginning of the sample. It is, therefore, interesting to examine the limiting distribution of the *CADF* test provided by Pesaran *et al.* (2013) and the consequence of its finite sample performance when Fourier form breaks appear in the DGP.

Theorem 3. Suppose Assumptions 1–6 hold and z_{it} is generated based on equation (9). Let $t_i^{PSY,B}(N,T)$ be the statistic for testing the unit-root hypothesis when Fourier form breaks exist in the DGP, and it is the t-statistic of $\hat{b}_i$ in the following cross-sectionally augmented Dickey–Fuller regression: $\Delta y_{it} = c_{i,0} + \boldsymbol{c}_{i,3}'\bar{\boldsymbol{z}}_{t-1} + \boldsymbol{c}_{i,4}'\Delta\bar{\boldsymbol{z}}_t + b_i y_{i,t-1} + e_{it}$. Then, as $N \to \infty$ for any fixed $T > 2k + 4$, we have:

$$t_i^{PSY,B}(N,T) \xrightarrow{N} \frac{\frac{\boldsymbol{\varepsilon}_{iy}'\boldsymbol{s}_{iy,-1}}{\sigma_i^2 T} - \mathring{\boldsymbol{q}}_{iT}' \boldsymbol{\Upsilon}_{fT}^{-1} \mathring{\boldsymbol{h}}_{iT}}{J_1^{p*} \times J_2^{p*}} \oplus \frac{O(T^{-1/2})}{O(T^{-1/4})}, \tag{26}$$

where $\mathring{\boldsymbol{q}}_{iT} = \left(\frac{\varepsilon_{iy}'\boldsymbol{F}}{\sigma_i\sqrt{T}} \quad \frac{\varepsilon_{iy}'\boldsymbol{\tau}}{\sigma_i\sqrt{T}} \quad \frac{\varepsilon_{iy}'\boldsymbol{s}_{f,-1}}{\sigma_i T} \right)'$, $\mathring{\boldsymbol{h}}_{iT} = \left(\frac{\boldsymbol{s}_{iy,-1}'\boldsymbol{F}}{\sigma_i T^{3/2}} \quad \frac{\boldsymbol{s}_{iy,-1}'\boldsymbol{\tau}}{\sigma_i T^{3/2}} \quad \frac{\boldsymbol{s}_{iy,-1}'\boldsymbol{s}_{f,-1}}{\sigma_i T^2} \right)'$, $\boldsymbol{g}_{iT} = \begin{pmatrix} \mathring{\boldsymbol{q}}_{iT} \\ \frac{\boldsymbol{s}_{iy,-1}'\varepsilon_{iy}}{\sigma_i^2 T} \end{pmatrix}$,

$$\boldsymbol{\Upsilon}_{fT} = \begin{pmatrix} \frac{\boldsymbol{F}'\boldsymbol{F}}{T} & \frac{\boldsymbol{F}'\boldsymbol{\tau}}{T} & \frac{\boldsymbol{F}'\boldsymbol{s}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}'\boldsymbol{F}}{T} & 1 & \frac{\boldsymbol{\tau}'\boldsymbol{s}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{s}_{f,-1}'\boldsymbol{F}}{T^{3/2}} & \frac{\boldsymbol{s}_{f,-1}'\boldsymbol{\tau}}{T^{3/2}} & \frac{\boldsymbol{s}_{f,-1}'\boldsymbol{s}_{f,-1}}{T^2} \end{pmatrix}, \boldsymbol{Q}_{iT} = \begin{pmatrix} \boldsymbol{\Upsilon}_{fT} & \mathring{\boldsymbol{h}}_{iT} \\ \mathring{\boldsymbol{h}}_{iT}' & \frac{\boldsymbol{s}_{iy,-1}'\boldsymbol{s}_{iy,-1}}{\sigma_i^2 T^2} \end{pmatrix}, J_1^{p*} = \left(\frac{\boldsymbol{\varepsilon}_{iy}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2(T-2k-4)} - \frac{\boldsymbol{g}_{iT}'\boldsymbol{Q}_{iT}^{-1}\boldsymbol{g}_{iT}}{T-2k-4}\right)^{1/2},$$

and $J_2^{p*} = \left(\frac{\boldsymbol{s}_{iy,-1}'\boldsymbol{s}_{iy,-1}}{\sigma_i^2 T^2} - \mathring{\boldsymbol{h}}_{iT}' \boldsymbol{\Upsilon}_{fT}^{-1} \mathring{\boldsymbol{h}}_{iT}\right)^{1/2}$. The notation '$\oplus$' is adapted from the Farey sequence denoting $(a/b) \oplus (c/d) = (a + c)/(b + d)$. If next to N, T also tends to infinity, then $t_i^{PSY,B}(N,T)$ has the following sequential limiting distribution:

$$t_i^{PSY,B}(N,T) \xrightarrow{(N,T)_{seq}} \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}_{i\mathbf{v}}'\boldsymbol{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}_{i\mathbf{v}}'\boldsymbol{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}, \tag{27}$$

where

$$\boldsymbol{\omega}_{i\mathbf{v}} = \begin{pmatrix} W_i(1) \\ \int_0^1 \left[\boldsymbol{W_f}(r)\right] dW_i(r) \end{pmatrix}, \boldsymbol{\pi}_{i\boldsymbol{v}} = \begin{pmatrix} \int_0^1 W_i(r)dr \\ \int_0^1 \left[\boldsymbol{W_f}(r)\right] W_i(r)dr \end{pmatrix},$$

$$\boldsymbol{G_v} = \begin{pmatrix} 1 & \int_0^1 \left[\boldsymbol{W_f}(r)\right]' dr \\ \int_0^1 \left[\boldsymbol{W_f}(r)\right] dr & \int_0^1 \left[\boldsymbol{W_f}(r)\right] \left[\boldsymbol{W_f}(r)\right]' dr \end{pmatrix}.$$

The right-hand side of equation (27) is the same as the limiting distribution of the *CADF* statistic proposed by Pesaran *et al.* (2013, Theorem 2.1) when there is no break in the DGP.

Proof: See Appendix S1.

The limiting distribution of $t_i^{PSY,B}(N,T)$ under a fixed T includes the bias terms $O_p(T^{-1/2})$ and $O_p(T^{-1/4})$, which are the finite-sample biases of the slope and standard error estimates respectively. These bias terms arise from omitting break terms in Pesaran's cross-sectionally augmented regression, but they disappear as T tends to infinity. However, due to the slow rate of convergence (in the Farey sums of $\oplus\frac{O(T^{-1/2})}{O(T^{-1/4})}$), these biases could be substantial even in finite T with infinite N when amplitude values are large. It is straightforward to show that under a fixed T, $O_p(T^{-1/4})$ is positive but $O_p(T^{-1/2})$ can be either positive or negative depending on the relative influence of the factor loadings ($\boldsymbol{\Gamma}_i$) and the parameters of Fourier terms ($\boldsymbol{A}_i$). Therefore, it is hard to predict the direction and magnitude of the size distortions in finite samples for Pesaran *et al.*'s (2013) *CIPS* test when smooth breaks appear in the DGP. A trivial fact from Theorem 3 is that the finite sample bias of the *CIPS* test is generally small when the amplitude of the breaks is small. However, the test may either seriously under- or over-reject the unit-root hypothesis in finite samples (T) when amplitude values are large. Our simulation results in Table 2 provide several cases to indicate that the size distortions of the *CIPS* test are serious under commonly used sample sizes ($T = 100$ and $T = 200$) when amplitude values are either medium or large.

BCADF-based panel unit-root tests

To develop a panel unit-root test, this paper considers the breaks and cross-sectional dependence augmented version of the *IPS* test (*BCIPS*):

$$BCIPS(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T), \tag{28}$$

and considers the mean deviation:

$$D(N,T) = N^{-1}\sum_{i=1}^{N}\left(t_i(N,T) - BCADF_{if}\right).$$

There is no guarantee that $D(N,T)=o_p(1)$ for N and T sufficiently large unless the $t_i(N,T)$ in equation (28) have finite moments for all N and T above some finite threshold values, say, N_0 and T_0. However, it is difficult to establish such moment conditions even under the case with cross-sectionally independent observations (IPS, 2003).

Following Pesaran (2007) and Pesaran *et al.* (2013), we construct the truncated version of the *BCIPS* statistic:

$$BCIPS^{*}(N,T)=\frac{1}{N}\sum_{i=1}^{N}t_i^{*}(N,T), \tag{29}$$

where

$$\begin{aligned} t_i^{*}(N,T)&=t_i(N,T), \quad \text{if } -M_1<t_i(N,T)<M_2,\\ t_i^{*}(N,T)&=-M_1, \quad \text{if } t_i(N,T)\leqslant -M_1,\\ t_i^{*}(N,T)&=M_2, \quad \text{if } t_i(N,T)\geqslant M_2. \end{aligned}$$

M_1 and M_2 are two positive constants such that $Pr(-M_1<t_i(N,T)<M_2)$ is sufficiently large.[5] Following the arguments in Pesaran *et al.* (2013), we can show that $BCIPS^{*}(N,T)$ converges almost surely to a distribution that is free of nuisance parameters.[6] The distributions of the *BCIPS* statistic and its truncated counterpart, $BCIPS^{*}$, are non-standard even for sufficiently large N. This is due to the dependence of the individual $BCADF_{if}$ on the common process $\boldsymbol{W_f}(r)$, invalidating the application of the standard central limit theorem to *BCIPS* or $BCIPS^{*}$. Our results are in contrast to those of IPS under cross-sectional independence, where a standardized version of *ADF* was shown to be normally distributed for N sufficiently large. Although the limiting distribution of $BCIPS^{*}(N,T)$ is not analytically tractable, it can be readily simulated by using equation (29).

Unit-root tests in the presence of a single factor

If there is only a single factor in the DGP, i.e. $m=1$ and $\boldsymbol{f}_t=f_t$ in equation (1), as that of Pesaran (2007), no additional variable ($\boldsymbol{x}_{it}$) is needed to approximate the unobservable factor, i.e. $\bar{z}_{t-1}=\bar{y}_{t-1}$ and $\Delta\bar{z}_t=\Delta\bar{y}_t$. In such a case, the rank condition in Assumption 5 requires that $\bar{\gamma}=\frac{1}{N}\sum_{i=1}^{N}\gamma_{iy}\neq 0$ and that f_t can be measured by a linear combination of $\sin(2\pi\kappa t/T)$, $\cos(2\pi\kappa t/T)$, $\Delta\bar{y}_t$ and $\bar{y}_{t-1}$. We therefore regress the following breaks and cross dependence augmented Dickey–Fuller equation using *OLS*:

$$\begin{aligned}\Delta y_{it}=&\,c_{i,0}+c_{i,1}\sin(2\pi\kappa t/T)+c_{i,2}\cos(2\pi\kappa t/T)+c_{i,3}\bar{y}_{t-1}\\ &+c_{i,4}\Delta\bar{y}_t+b_i y_{i,t-1}+e_{it}.\end{aligned} \tag{30}$$

The t-statistic of the estimate of $b_i(\hat{b}_i)$ in equation (30) can be expressed as:

[5] The construction of M_1 and M_2 is described in Pesaran (2007).

[6] This distribution depends on M_1, M_2 and $\boldsymbol{W_f}(r)$. The included Fourier terms are deterministic functions which only affect the conditional expectation of $CADF_{if}^{*}$ in Pesaran *et al.* (2013), i.e. $E(CADF_{1f}|\boldsymbol{W_f})$. It is, therefore, appropriate to discuss the convergence of $BCIPS^{*}(N,T)$ by following their arguments for the convergence of $CIPS^{*}(N,T)$.

$$t_i^{\diamondsuit}(N,T) = \frac{\Delta \boldsymbol{y}_i' \boldsymbol{M}_z^{\diamondsuit} \boldsymbol{y}_{i,-1}}{\tilde{\sigma}_i (\boldsymbol{y}_{i,-1}' \boldsymbol{M}_z^{\diamondsuit} \boldsymbol{y}_{i,-1})^{1/2}}, \tag{31}$$

where $\boldsymbol{M}_z^{\diamondsuit} = \boldsymbol{I}_T - \boldsymbol{Z}^{\diamondsuit}(\boldsymbol{Z}'^{\diamondsuit}\boldsymbol{Z}^{\diamondsuit})^{-1}\boldsymbol{Z}'^{\diamondsuit}$, $\boldsymbol{Z}^{\diamondsuit} = (\Delta\bar{\boldsymbol{y}}, \boldsymbol{\tau}, \boldsymbol{\Upsilon}_1, \boldsymbol{\Upsilon}_2, \bar{\boldsymbol{y}}_{-1})$, $\Delta\bar{\boldsymbol{y}} = (\Delta\bar{y}_1, \Delta\bar{y}_2, \ldots, \Delta\bar{y}_T)'$, $\bar{\boldsymbol{y}}_{-1} = (\bar{y}_0, \bar{y}_1, \ldots, \bar{y}_{T-1})'$, $\tilde{\sigma}_i^2 = \frac{\Delta \boldsymbol{y}_i' \boldsymbol{M}_{i,z}^{\diamondsuit} \Delta \boldsymbol{y}_i}{T-6}$, $\boldsymbol{M}_{i,z}^{\diamondsuit} = \boldsymbol{I}_T - \boldsymbol{G}_i^{\diamondsuit}(\boldsymbol{G}_i'^{\diamondsuit}\boldsymbol{G}_i^{\diamondsuit})^{-1}\boldsymbol{G}_i'^{\diamondsuit}$ and $\boldsymbol{G}_i^{\diamondsuit} = (\boldsymbol{Z}^{\diamondsuit}, \boldsymbol{y}_{i,-1})$. The limiting distribution of $t_i^{\diamondsuit}(N,T)$ under the null hypothesis is obtained from the results in Theorem 2 by replacing the m-dimensional standard Brownian motion, $\boldsymbol{W_f}(r)$, with a scalar independent standard Brownian motion, $W_f(r)$.

De Silva, Hadri and Tremayne (2009) point out two conditions for Pesaran's (2007) single factor *CIPS* (or *CADF*) test to perform adequately. First, there is only one single factor in the model. Second, the average of the factor loadings (across i) needs to be different from zero, i.e. $\bar{\gamma} = \frac{1}{N}\sum_{i=1}^{N} \gamma_{iy} \neq 0$. Furthermore, they criticize that Pesaran's (2007) single factor *CIPS* test displays poor empirical performance when these conditions are not satisfied.[7] The same criticism is expected to hold in the single-factor Fourier form panel unit-root test. By allowing for multiple factors in the model, our multi-factor *BCIPS* tests are free from the criticism of assuming $\bar{\gamma} \neq 0$.

The case with serially correlated errors

Our discussion in section 'Unit-root tests in the presence of multiple factors' can be extended to the case where individual-specific errors are serially correlated. Following Pesaran (2007), two different specifications for serially correlated errors are given as follows:

$$u_{it} = \rho_i u_{i,t-1} + \eta_{iyt}, \quad \eta_{iyt} = \boldsymbol{\gamma}_{iy}' \boldsymbol{f}_t + \varepsilon_{iyt}; \tag{32}$$

$$u_{it} = \boldsymbol{\gamma}_{iy}' \boldsymbol{f}_t + \eta_{iyt}, \quad \eta_{iyt} = \rho_i \eta_{iy,t-1} + \varepsilon_{iyt}, \tag{33}$$

where ε_{iyt} is an idiosyncratic error. We focus our discussion on the specification of equation (33). The asymptotic distribution to be derived in this section can be adapted to deal with both specifications in equations (32) and (33). By replacing u_{it} in equation (1) with (33), equation (3) can be rewritten as:

$$\Delta y_{it} = \beta_i y_{i,t-1} - \beta_i \boldsymbol{\alpha}_{iy}' \boldsymbol{d}_t + \phi_i \boldsymbol{\alpha}_{iy}' \Delta \boldsymbol{d}_t + \boldsymbol{\gamma}_{iy}' \boldsymbol{f}_t + \eta_{iyt}, \quad t = 1, .., T, \; i = 1, .., N. \tag{34}$$

We assume the coefficient ρ_i in equation (33) to be homogeneous across i, but it could be relaxed at the cost of more complex mathematical details. Under the null hypothesis that $\beta_i = 0$, with $\rho_i = \rho$, equation (34) becomes:

$$\Delta y_{it} = \rho \Delta y_{i,t-1} + \boldsymbol{\alpha}_{iy}'(\Delta \boldsymbol{d}_t - \rho \Delta \boldsymbol{d}_{t-1}) + \boldsymbol{\gamma}_{iy}'(\boldsymbol{f}_t - \rho \boldsymbol{f}_{t-1}) + \varepsilon_{iyt}. \tag{35}$$

To test the null hypothesis in equation (4), this paper estimates the following breaks and cross-sectional dependence augmented ADF regression (*BCADF*) for each individual:

$$\begin{aligned} \Delta y_{it} = {} & c_{i,0} + c_{i,1}\sin(2\pi\kappa t/T) + c_{i,2}\cos(2\pi\kappa t/T) + \boldsymbol{c}_{i,3}' \bar{\boldsymbol{z}}_{t-1} \\ & + \boldsymbol{c}_{i,4}' \Delta \bar{\boldsymbol{z}}_t + \boldsymbol{c}_{i,5}' \Delta \bar{\boldsymbol{z}}_{t-1} + c_{i,6} \Delta y_{i,t-1} + b_i y_{i,t-1} + e_{it}, \quad t = 1, 2, \ldots, T. \end{aligned} \tag{36}$$

[7] We are grateful to a reviewer for bringing this to our attention.

The t-statistic of the estimate of b_i ($\hat{b}_i$) is then applied to examine the unit-root hypothesis, and it can be written as:

$$t_i^{\rho}(N,T)=\frac{\Delta\boldsymbol{y}_i'\boldsymbol{M}_z^{\rho}\boldsymbol{y}_{i,-1}}{\hat{\sigma}_i(\boldsymbol{y}_{i,-1}'\boldsymbol{M}_z^{\rho}\boldsymbol{y}_{i,-1})^{1/2}}, \tag{37}$$

where $\hat{\sigma}_i^2=\frac{\Delta\boldsymbol{y}_i'\boldsymbol{M}_{i,z}^{\rho}\Delta\boldsymbol{y}_i}{T-(3k+6)}$, $\boldsymbol{M}_z^{\rho}=\boldsymbol{I}_T-\boldsymbol{Z}^{\rho}(\boldsymbol{Z}^{\rho\prime}\boldsymbol{Z}^{\rho})^{-1}\boldsymbol{Z}^{\rho\prime}$, $\boldsymbol{Z}^{\rho\prime}=(\Delta\boldsymbol{y}_{i,-1},\Delta\bar{z}_{-1},\Delta\bar{z},\boldsymbol{\tau},\boldsymbol{\Upsilon}_1,\boldsymbol{\Upsilon}_2,\bar{z}_{-1})$, $\boldsymbol{M}_{i,z}^{\rho}=\boldsymbol{I}_T-\boldsymbol{G}_i^{\rho}(\boldsymbol{G}_i^{\prime\rho}\boldsymbol{G}_i^{\rho})^{-1}\boldsymbol{G}_i^{\prime\rho}$ and $\boldsymbol{G}_i^{\rho}=(\boldsymbol{Z}^{\rho},\boldsymbol{y}_{i,-1})$. The limiting distribution of $t_i^{\rho}(N,T)$ does not depend on nuisance parameters as stated in the following theorem.

Theorem 4. Let z_{it} be generated based on equations (7) and (35) with the cross-sectional mean of the initial observation $\bar{z}_0$ being zero and $|\rho|<1$. Suppose that Assumptions 1–6 hold. Then $t_i^{\rho}(N,T)$ in equation (37) has the same sequential and joint limiting distribution, given by equation (22), as obtained under $\rho=0$.

Proof. See Appendix S1.

The *BCIPS* test can be applied to the case with serially correlated errors since $t_i^{\rho}(N,T)$ in equation (37) has the same limiting distribution as that of equation (22). The specification of the errors in equation (33) can be generalized to an $ARMA(l,s)$ process:

$$(1-\rho_{i,1}L-\cdots-\rho_{i,l}L^l)\eta_{iyt}=(1+\theta_{i,1}L+\cdots+\theta_{i,s}L^s)\varepsilon_{iyt},$$

in which all roots of $(1-\rho_{i,1}z-\cdots-\rho_{i,l}z^l)=0$ and $(1+\theta_{i,1}z+\cdots+\theta_{i,s}z^s)=0$ lie outside the unit circle. In such a case, we suggest the following *BCADF* regression:[8]

$$\begin{aligned}\Delta y_{it}&=c_{i,0}+c_{i,1}\sin(2\pi\kappa t/T)+c_{i,2}\cos(2\pi\kappa t/T)+\boldsymbol{c}_{i,3}'\bar{\boldsymbol{z}}_{t-1}+\boldsymbol{c}_{i,4}'\Delta\bar{\boldsymbol{z}}_t\\&\quad+\sum_{j=1}^{p}\boldsymbol{c}_{i,5,j}'\Delta\bar{\boldsymbol{z}}_{t-j}+\sum_{j=1}^{p}c_{i,6,j}\Delta y_{i,t-j}+b_iy_{i,t-1}+e_{it},\quad t=1,2,\ldots,T,\end{aligned} \tag{38}$$

where the value for the lagged order p is chosen to ensure that there is no remaining serial correlation in the residuals.[9] It is easily seen that the limiting distribution of $t_i^{\rho}(N,T)$ with $N\to\infty$ for a fixed T depends on the lag augmentation order p in the regression. We, therefore, construct critical values of $t_i^{\rho}(N,T)$ for different values of p.

Remark 2. The homogeneity assumptions on the Fourier frequencies and the lag orders of the model across individuals, inherited from Pesaran *et al.* (2013), are restrictive. A feasible procedure to relax the above homogeneity assumptions is to apply the de-factor method in the PANIC (panel analysis of non-stationarity in the idiosyncratic and common components) proposed by Bai and Ng (2004). The sketch of this procedure is given in Appendix S1. However, in the case without structural breaks, Pesaran *et al.* (2009, 2012) show that their proposed tests have correct sizes for all combinations of N and T, but the tests proposed by Bai and Ng (2004) over-reject the null hypothesis in many cases,

[8] This is also based on the assumption that $\rho_{i,j}=\rho_j, j=1,\ldots,l$ and $\theta_{i,j}=\theta_j, j=1,\ldots,s, \forall i=1,\ldots,N$.

[9] It is necessary to let p be a function of T and N to ensure consistent estimates in equation (38). (See e.g. Said and Dickey (1984) and Bai and Ng (2004)). The detailed derivation of this condition poses additional technical difficulties and will not be pursued here.

especially when the model includes a linear trend. The suggested procedure in this remark is expected to suffer the same size problem too.

Critical values of the *BCADF* test for different values of N, T, κ, k and p are obtained by stochastic simulation. The asymptotic distribution of $t_i^{\rho}(N,T)$ depends only on the Fourier frequency, κ, but is invariant to $\boldsymbol{A}_i$, $\boldsymbol{\Gamma}_i$, $\boldsymbol{\Psi}(L)$ or σ_i. Without loss of generality, we set $\boldsymbol{A}_i = 0$, $\boldsymbol{\Gamma}_i = 0$, $\boldsymbol{\Psi}(L) = \boldsymbol{I}$ and $\sigma_i = \sigma = 1$.

To simulate the critical values of the *BCIPS* statistic, the series of y_{it}s are generated by $y_{it} = y_{i,t-1} + \varepsilon_{iyt}$ for $i = 1,\ldots, N$, and $t = 1, 2,\ldots, T$ with $y_{i0} \sim$ i.i.d.$N(0,1)$. The jth element of the $k \times 1$ vector of additional regressors, x_{ijt}, is generated based on $x_{ijt} = x_{ij,t-1} + \varepsilon_{ixjt}$, $i = 1,\ldots, N; j = 1, 2,\ldots, k; t = 1, 2,\ldots, T$ with $x_{ij,0} \sim$ i.i.d.$N(0,1)$.[10] Here ε_{iyt} and ε_{ixjt} are both i.i.d.$N(0,1)$ and independent of each other. After generating y_{it} and x_{ijt}, we regress Δy_{it} on an intercept, $\sin(2\pi\kappa t/T)$, $\cos(2\pi\kappa t/T)$, $\bar{z}'_{t-1}$, $[\Delta\bar{z}'_t,\ldots, \Delta\bar{z}'_{t-p}]$, $[\Delta y_{i,t-1},\ldots, \Delta y_{i,t-p}]$ and $y_{i,t-1}$ over the frequency $\kappa = 1,\ldots, 5$ and the sample $t = 1,\ldots, T$. The $t_i(N,T)$ statistic is the t-ratio of the coefficient on $y_{i,t-1}$. The *BCIPS* statistic is then computed based on equation (28). Critical values of the *BCADF* and *BCIPS* statistics can be simulated by repeating the above procedures 10,000 times. The main focus of the paper is to develop panel unit-root tests, and hence we consider the results from the individual *BCADF* test as secondary to the corresponding results from the panel *BCIPS* test. We therefore do not report the critical values of the *BCADF* statistic to save space, but they are available from the authors upon request.

The 1%, 5% and 10% critical values of the *BCIPS* statistic for the model with an intercept only and for the model with an intercept and a linear trend, under different κ, k, p, N and T, are reported in Tables S1–S8.[11] If the critical values of the *BCIPS** and *BCIPS* statistics are different, then the value of the former statistic is slightly larger than that of the latter. This indicates a slightly rightward shift of the null distribution of the *BCIPS** statistic relative to that of the *BCIPS* statistic. To save space, the critical values of the *BCIPS** statistic are not reported, but they are available from the authors upon request.

A data-driven method of selecting κ and p

Empirically, we do not know the values of the Fourier frequency (κ) and the lag order (p) of the model, and hence they need to be determined first. We modify Enders and Lee's (2012a) grid-search method to determine κ and p jointly. To be specific, this paper sets the maximum Fourier frequency parameter, κ^{max}, and the maximum lag order of the model, p^{max}, to 5 and 4 respectively and then estimates equation (38) for different lag orders, $p = 0, 1,\ldots, 4$, under a given κ. We apply the *SBC* rule to determine the optimal lag order $\hat{p}$ and then construct $SSR_{\kappa,\hat{p}}$ under a given N, T and κ. The *SBC* under a given κ is:

$$SBC = \frac{-TN}{2}(1 + \ln 2\pi) - \frac{T}{2}\sum_{i=1}^{N} \ln\left(\frac{\sum_{t=1}^{T}\hat{e}_{it}^2}{T}\right), \tag{39}$$

[10] No additional regressor, x_{it}, is included for the case with a single factor and the DGP of y_{it} is: $y_{it} = y_{i,t-1} + f_t + \varepsilon_{iyt}$.

[11] The critical values of the *BCIPS* test for the model without an intercept and a linear trend are available from the authors upon request.

where $\hat{e}_{it}$ is the residual estimate in equation (38). The optimal κ is obtained by minimizing the sum of squared residuals, $SSR_{\kappa,\hat{p}}$, across different values of κ: $\hat{\kappa} = \arg\min_{\kappa} SSR_{\kappa,\hat{p}}$. Based on $\hat{\kappa}$ and $\hat{p}$, the *BCIPS* statistic is calculated and the associated critical value is applied. Applying the above method to determine $\hat{\kappa}$ and $\hat{p}$, the sizes of the *BCIPS* test are reasonable for $T \geqslant 100$ under a known two-factor model as discussed in section 'Test with κ and p unknown'.

Uncertainty about the number of factors

Although it is reasonable to assume that the number of factors m is bounded by a sufficiently large integer, m_{max}, it is unknown in practice. Following Pesaran *et al.* (2013), there are two possible methods to proceed with the proposed test when m is unknown. The first one is to set $k = m_{max} - 1$ if there exist k additional regressors to augment the *BCADF* regression. In this case, the true number of factors is allowed to be any integer value between one and m_{max}. The second one is to estimate m consistently by a suitable statistical technique such as the information criteria proposed by Bai and Ng (2002) and Moon and Perron (2004). With the estimated number of factors $\hat{m}$, the number of additional variables for augmentation is $k = \hat{m} - 1$.

IV. Finite sample performance

To examine the finite sample properties of the *BCIPS* test, this paper focuses on the case with two factors.[12] The data generating process is therefore given as follows:

$$y_{it} = \mu_{iy}(1-\phi_i L)t + (1-\phi_i L)\varpi_{i,\kappa,t} + \phi_i y_{i,t-1} + u_{it}, \quad i=1,2,\dots,N,\ t=1,2,..,T,$$

where $\varpi_{i,\kappa,t} = \mu_i + \alpha_{iy,1}\sin(2\pi\kappa t/T) + \alpha_{iy,2}\cos(2\pi\kappa t/T)$; $u_{it} = \gamma_{iy,1}f_{1t} + \gamma_{iy,2}f_{2t} + \eta_{iy,t}$, $\eta_{iy,t} = \rho_{iy}\eta_{iy,t-1} + (1-\rho_{iy}^2)^{1/2}\varepsilon_{iyt}$, $y_{i0}, \varepsilon_{iy0} \sim$ i.i.d.$N(0,1)$. Following Pesaran *et al.* (2013), we set $f_{1t}, f_{2t} \sim$ i.i.d.$N(0,1)$, $\gamma_{iy,1} \sim$ i.i.d.$U[0,2]$, $\gamma_{iy,2} \sim$ i.i.d.$U[0,1]$, $\varepsilon_{iyt} \sim$ i.i.d.$N(0,\sigma_i^2)$ with $\sigma_i^2 \sim$ i.i.d.$U[0.5,1.5]$, and $\rho_{iy} \sim$ i.i.d.$U[0.2,0.4]$ and i.i.d.$U[-0.4,-0.2]$ to denote the cases of positive and negative residual serial correlation, respectively. We consider different magnitudes for amplitude parameters: $\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U[1,2]$, $\sim$ i.i.d.$U[10,100]$, and $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U[1,2]$, $\sim$ i.i.d.$U[3,5]$, $\sim$ i.i.d.$U[10,20]$. These are examples of medium and large amplitude values and of opposite sign in amplitude coefficients. One additional regressor, x_{it}, is generated by $\Delta x_{it} = d_{ix} + \alpha_{ix,1}\Delta\sin(2\pi\kappa t/T) + \alpha_{ix,2}\Delta\cos(2\pi\kappa t/T) + \gamma_{ix,1}f_{1t} + \gamma_{ix,2}f_{2t} + \eta_{ix,t}$, $\eta_{ix,t} = \rho_{ix}\eta_{ix,t-1} + \varepsilon_{ixt}$, where $x_{i,0} \sim$ i.i.d.$N(0,1)$, $\gamma_{ix,1} \sim$ i.i.d.$U[0,2]$, $\gamma_{ix,2} = 0$,[13] $\rho_{ix} \sim$ i.i.d.$U[0.2,0.4]$, and $\varepsilon_{ixt} \sim$ i.i.d.$N(0,1-\rho_{ix}^2)$. The amplitude parameters are set as: $\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[1,2]$, $\sim$ i.i.d.$U[3,5]$ and $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[1,2]$, $\sim$ i.i.d.$U[3,5]$. For the intercept case, $\mu_i \sim$ i.i.d.$N(1,1)$, $\mu_{iy} = 0$ and $d_{ix} = 0$. As for the linear trend case, $\mu_i \sim$ i.i.d.$N(0,0.02)$, $d_{ix} = \delta_i$ and $\mu_{iy}, \mu_i, \delta_i \sim$ i.i.d.$U[0,0.02]$.

[12] The sizes and powers of the $BCIPS^*$ statistic are the same as those of *BCIPS* for $T \geqslant 50$, and they are available from the authors upon request.

[13] The factor loadings are generated so that $E(\mathbf{\Gamma}_i) = \begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$ satisfies the rank condition (16). The same assumptions for the first and second factor loadings can also be found in Pesaran *et al.* (2013).

Sizes are computed under the null hypothesis of $\phi_i = 1$ for all i. Powers are constructed under the alternative hypothesis of $\phi_i \sim$ i.i.d.$U[0.85, 0.95]$. The common factors (f_{1t}, f_{2t}) were generated independently of ε_{it}, and the parameters $\phi_i, \mu_i, \mu_{iy}, \alpha_{iy,1}, \alpha_{iy,2}, \alpha_{ix,1}, \alpha_{ix,2}, \gamma_{iy,1}, \gamma_{iy,2}, \gamma_{ix,1}, \rho_{ix}, \rho_{iy}, d_{ix}$ *and* σ_i were also drawn independently of ε_{it}. The tests were one-sided with the nominal size set at 5% and were conducted for $N = 20, 30, 50, 100, 200$, $T = 50, 70, 100, 200$ and $\kappa = 1, 2, 3$. The size and power for each experiment were constructed using 2,000 replications. Critical values for different combinations of κ, p, k, N and T under the model with an intercept and the model with an intercept and a linear trend, reported in Appendix S2 (Tables B1–B8), are adopted to examine the size and power of the *BCIPS* statistic. To save space, only the finite sample properties of the *BCIPS* test based on the former model are reported, and the results based on the latter model are reported in Appendix S2.

Size and power when factors and idiosyncratic errors are serially uncorrelated

As a benchmark, we assume that the frequency parameter, κ, is known in both the DGP and the regression, but the lag order of the model, p, is known in the DGP but unknown in the regression. It is determined by the *SBC* rule in equation (39) under different values of κ. The size of the test with an unknown κ and p is examined in section 'Test with κ and p unknown'. We consider three different magnitudes for the amplitude coefficients. They are $\alpha_{iy,j}, \alpha_{ix,j} \sim$ i.i.d.$U[1, 2], j = 1, 2$, (case A), $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U[10, 20]$, $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[3, 5]$ (case B), and $\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U[10, 100], \alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[3, 5]$ (case C). Table 1 indicates that the sizes are generally close to 0.05 regardless of amplitude values. The above results agree with Theorems 1 and 2, indicating that the limiting distribution of the *BCADF* statistic does not depend on nuisance parameters. Similar results are also obtained when the model with an intercept and a linear trend is adopted, as indicated by Table B9 in Appendix S2.

The last three panels in Table 1 point out that the powers of the *BCIPS* test are generally greater than 0.5 for $T \geqslant 50$ when using the different amplitude values in cases A, B and C. Under a given N and κ, the power of the *BCIPS* test increases with T significantly and is close to 1.0 for most cases when $T \geqslant 100$. This implies that the *BCIPS* test is consistent. The power also increases with κ when T and N are given, which is consistent with the results of Enders and Lee (2012a, Table 3).[14] Similar results are observed when the model with an intercept and a linear trend is applied except that the powers of the *BCIPS* test are generally high when $T \geqslant 100$ (instead of $T \geqslant 50$) as indicated in Table B9 in Appendix S2. The above results indicate that the *BCIPS* test does not depend on nuisance parameters.

We next discuss the size distortion of the *CIPS* test by Pesaran *et al.* (2013) when smoothing breaks exist in the DGP. The lag order of the model is also selected based on the *SBC* rule in equation (39). This paper provides several cases to indicate that amplitude

[14] The intuition behind this result is that as this deterministic component moves away from the zero frequency, the persistence effect will diminish. A time series with high frequencies will be less persistent, and the power of unit root tests tends to increase. We appreciate a reviewer's suggestion. We also examine the power of the *BCIPS* test with multiple Fourier frequencies. The simulation results (available from the authors upon request) indicate that the power decreases as the number of Fourier frequencies increases, especially when the model includes a linear trend. These results reflect that an over-fitting phenomenon occurs when the number of Fourier terms increases (Enders and Lee, 2012a). As such, we recommend a single Fourier frequency unless T and N are large.

TABLE 1

Sizes and powers of the BCIPS test with two known factors ($m = 2$) in which factors and idiosyncratic errors are serially uncorrelated – with an Intercept only

	$\kappa = 1$					$\kappa = 2$					$\kappa = 3$				
$T \backslash N$	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200
Size															
$BCIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U[1,2]$, $\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[1,2]$															
50	0.053	0.039	0.048	0.045	0.044	0.044	0.050	0.056	0.046	0.047	0.053	0.054	0.043	0.052	0.048
70	0.049	0.038	0.044	0.042	0.052	0.045	0.040	0.045	0.046	0.055	0.048	0.053	0.053	0.050	0.044
100	0.047	0.050	0.038	0.046	0.049	0.046	0.049	0.051	0.045	0.039	0.038	0.054	0.045	0.048	0.047
200	0.056	0.034	0.047	0.044	0.043	0.042	0.047	0.046	0.046	0.047	0.048	0.044	0.049	0.052	0.051
$BCIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U[10,20]$, $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[3,5]$															
50	0.047	0.056	0.048	0.042	0.060	0.054	0.050	0.048	0.048	0.047	0.054	0.049	0.046	0.045	0.046
70	0.043	0.056	0.049	0.050	0.053	0.048	0.056	0.047	0.044	0.057	0.050	0.046	0.055	0.048	0.051
100	0.038	0.039	0.046	0.051	0.048	0.047	0.046	0.050	0.049	0.056	0.056	0.043	0.052	0.051	0.052
200	0.051	0.046	0.043	0.047	0.052	0.047	0.051	0.046	0.044	0.047	0.039	0.053	0.052	0.043	0.047
$BCIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U[10,100]$, $\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[3,5]$															
50	0.041	0.050	0.046	0.050	0.042	0.045	0.049	0.060	0.068	0.054	0.057	0.045	0.047	0.048	0.049
70	0.044	0.039	0.055	0.057	0.054	0.053	0.044	0.046	0.045	0.046	0.047	0.059	0.054	0.053	0.052
100	0.049	0.049	0.040	0.045	0.055	0.047	0.044	0.051	0.045	0.039	0.041	0.040	0.053	0.047	0.045
200	0.051	0.040	0.048	0.047	0.043	0.053	0.046	0.054	0.051	0.048	0.063	0.045	0.045	0.050	0.048
Power															
$BCIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U[1,2]$, $\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[1,2]$															
50	0.406	0.436	0.541	0.546	0.618	0.556	0.555	0.629	0.615	0.662	0.662	0.651	0.729	0.799	0.846
70	0.579	0.752	0.793	0.924	0.930	0.736	0.866	0.910	0.955	0.957	0.830	0.943	0.975	0.996	0.997
100	0.881	0.983	0.980	1.000	1.000	0.967	0.998	0.998	1.000	1.000	0.990	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(*continued*)

TABLE 1

(*Continued*)

$T \backslash N$	$\kappa = 1$ 20	30	50	100	200	$\kappa = 2$ 20	30	50	100	200	$\kappa = 3$ 20	30	50	100	200
$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U[10,20]$, $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[3,5]$															
50	0.367	0.492	0.559	0.654	0.764	0.481	0.560	0.632	0.676	0.700	0.581	0.707	0.764	0.866	0.926
70	0.549	0.756	0.815	0.956	0.988	0.725	0.901	0.921	0.979	0.995	0.866	0.966	0.988	0.999	1.000
100	0.969	0.941	0.998	1.000	1.000	0.997	0.985	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U[10,100]$, $\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U[3,5]$															
50	0.218	0.354	0.319	0.284	0.301	0.382	0.603	0.659	0.697	0.693	0.598	0.819	0.906	0.969	0.990
70	0.466	0.617	0.692	0.873	0.949	0.841	0.937	0.965	0.998	1.000	0.950	0.990	0.999	1.000	1.000
100	0.966	0.987	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: y_{it} is generated as $y_{it} = (1-\phi_i L)(\mu_i + \alpha_{iy,1}\sin(2\pi kt/T) + \alpha_{iy,2}\cos(2\pi kt/T)) + \phi_i y_{it-1} + \gamma_{iy,1} f_{1t} + \gamma_{iy,1} f_{2t} + \eta_{iyt}$, $\eta_{iyt} = \rho_{iy}\eta_{iyt-1} + (1-\rho_{iy}^2)^{1/2}\varepsilon_{iyt}$, with y_{i0}, $\varepsilon_{iy0} \sim$ i.i.d.$(0,1)$, $\mu_i \sim$ i.i.d.$N[1,1]$, $\gamma_{iy,1} \sim$ i.i.d.$U[0,2]$, $\gamma_{iy,2} \sim$ i.i.d.$U[0,1]$, $f_{1t}, f_{2t} \sim$ i.i.d.$N(0,1)$, $\varepsilon_{iyt} \sim$ i.i.d.$N(0,\sigma_i^2)$ with $\sigma_i^2 \sim$ i.i.d.$U[0.5,1.5]$; $\rho_{iy} \sim$ i.i.d.$U[0.2,0.4]$ and $\sim$ i.i.d.$U[-0.4,-0.2]$ to denote the case of positive and negative residual serial correlation respectively. $x_{it} = x_{it-1} + \alpha_{ix,1}\Delta\sin(2\pi kt/T) + \alpha_{ix2}\Delta\cos(2\pi\kappa t/T) + \gamma_{ix,1} f_{1t} + \eta_{ixt}$, $\eta_{ixt} = \rho_{ix}\eta_{it-1} + \varepsilon_{ixt}$, $x_{i0} \sim$ i.i.d.$N(0,1)$, $\gamma_{ix,1} \sim$ i.i.d.$U[0,2]$, $\rho_{ix} \sim$ i.i.d.$U[0.2,0.4]$ and $\varepsilon_{ixt} \sim$ i.i.d.$N(0,1-\rho_{ix}^2)$. Sizes (under the null $\phi_i = 1$) and Powers (under the alternative $\phi_i \sim$ i.i.d.$U[0.85,0.95]$) of the *BCIPS* statistic are computed at the 5% nominal level based on the *BCADF* regression equation. The lag order of the model is selected based on the SBC of the panel: $SBC = -\frac{TN}{2}(1+\ln 2\pi) - \frac{T}{2}\sum_{i=1}^{N}\ln((\sum_{t=1}^{T}\hat{e}_{it}^2)/T)$, where T is the number of observations and N is the panel size. The *BCIPS* statistics is described by equation (28).

values affect the size of the *CIPS* test under different values of the frequency parameter (κ). They are $\alpha_{iy,1}, -\alpha_{iy,2}, -\alpha_{ix,1}, \alpha_{ix,2} \sim \text{i.i.d.}U[1,2]$ (case D) and $\sim \text{i.i.d.}U[3,5]$ (case E), and $\alpha_{iy,j}, \alpha_{ix,j}$ from case B for $j = 1,2$.[15] The results from Table 2 indicate that the *CIPS* test is oversized at $\kappa = 1$ even when amplitude values are medium (case D). By increasing amplitude values, the *CIPS* test generally reveals serious oversize distortions at $\kappa = 1$, mild oversize distortions at $\kappa = 2$ and serious under-size distortions when $\kappa > 2$, as indicated by the second and third panels of Table 2. In general, the oversize distortions of the *CIPS* test decrease with κ regardless of amplitude values. Based on the linear trend model, the results from Table B10 in Appendix S2 reveal that the *CIPS* test suffers serious oversize distortions with large amplitude values when $\kappa \leqslant 2$. The above results indicate that it may not be appropriate to apply the *CIPS* test in empirical applications when smoothing breaks in deterministic terms exist in data.

Size and power when factors are serially uncorrelated but idiosyncratic errors are serially correlated

In the case with first-order autoregressive errors, we consider the scenarios of positive and negative serial correlations with amplitude parameters being drawn from cases A (Table 3), B and C (Table 4), respectively. Tables 3 and 4 indicate that the sizes of the *BCIPS* test are close to 0.05 and the powers of the test are generally reasonable when $T > 50$ for both models regardless of N, κ and the sign of residual serial correlation. As for the powers of the *BCIPS* test, they are reasonably high in general when $T \geqslant 100$. Besides, the sizes of the *BCIPS* test under a positive residual serial correlation are generally smaller than those under a negative residual serial correlation, and the powers of the test increase with κ and T respectively. Similar results are obtained if the model with an intercept and a linear trend is applied, as indicated by Tables B11 and B12 in Appendix S2. The above results again support that the values of $\alpha_{iy,j}$ and $\alpha_{ix,j}$, $\forall j = 1,2$, in the Fourier function have little effect on the sizes and powers of the *BCIPS* test.

Test with κ and p unknown

This section examines the sizes of the *BCIPS* statistic when κ and p are known in the DGP but are unknown in the regression, and hence they are jointly determined based on the method discussed in section 'A data-driven method of selecting κ and p'. We focus our discussion on the case with break amplitudes being drawn from case A since break amplitudes in the Fourier function have little effect on the finite sample properties of the test, as discussed in sections 'Size and power when factors and idiosyncratic errors are serially uncorrelated' and 'Size and power when factors are serially uncorrelated but idiosyncratic errors are serially correlated'. The sizes of the *BCIPS* test under different N, T and κ are reported in Table 5 for the model with an intercept and in Appendix S2 (Table B13) for the model with an intercept and a linear trend. With serially uncorrelated residuals, the sizes of the *BCIPS* test are generally reasonable and close to 0.05 for the former model with $T > 50$ and for the latter one with $T \geqslant 100$. Although the sizes of the *BCIPS* test based on

[15] The sizes and powers of the *CIPS* test are reasonable when the amplitude values are small such as $\alpha_{iy,j}, \alpha_{ix,j} \sim \text{i.i.d.}U[0, 0.2]$ for $j = 1,2$. The results are available upon request from the authors.

TABLE 2

Sizes of Pesaran et al.'s (2013) CIPS test with two known factors in which factors and idiosyncratic errors are serially uncorrelated with an Intercept only

	$\kappa = 1$					$\kappa = 2$					$\kappa = 3$				
T\N	*20*	*30*	*50*	*100*	*200*	*20*	*30*	*50*	*100*	*200*	*20*	*30*	*50*	*100*	*200*
Size: Pesaran's $CIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U(1,2)$, $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U(1,2)$															
50	0.166	0.169	0.205	0.226	0.252	0.073	0.069	0.069	0.088	0.078	0.022	0.018	0.013	0.008	0.007
70	0.104	0.117	0.140	0.171	0.189	0.050	0.053	0.048	0.069	0.062	0.014	0.010	0.013	0.013	0.007
100	0.084	0.086	0.121	0.145	0.123	0.042	0.039	0.057	0.052	0.050	0.021	0.011	0.022	0.014	0.010
200	0.074	0.077	0.069	0.076	0.085	0.034	0.045	0.032	0.027	0.023	0.023	0.019	0.015	0.005	0.006
Size: Pesaran's $CIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U(3,5)$, $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U(3,5)$															
50	0.373	0.418	0.428	0.523	0.574	0.104	0.089	0.083	0.105	0.109	0.021	0.013	0.007	0.012	0.005
70	0.257	0.312	0.328	0.410	0.466	0.072	0.070	0.083	0.089	0.099	0.012	0.013	0.010	0.013	0.006
100	0.204	0.274	0.280	0.327	0.355	0.060	0.113	0.073	0.090	0.105	0.019	0.026	0.015	0.012	0.014
200	0.197	0.159	0.204	0.240	0.237	0.119	0.057	0.097	0.093	0.099	0.053	0.016	0.030	0.030	0.022
Size: Pesaran's $CIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U(10,20)$, $-\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U(3,5)$															
50	0.338	0.341	0.620	0.595	0.717	0.064	0.072	0.105	0.094	0.125	0.014	0.013	0.010	0.010	0.006
70	0.309	0.399	0.571	0.447	0.515	0.056	0.078	0.092	0.080	0.065	0.013	0.014	0.014	0.007	0.003
100	0.165	0.364	0.309	0.421	0.518	0.038	0.094	0.041	0.063	0.101	0.010	0.024	0.008	0.008	0.010
200	0.156	0.403	0.289	0.375	0.474	0.039	0.194	0.086	0.148	0.148	0.012	0.052	0.018	0.034	0.028

Notes: Same as those in Table 1.

TABLE 3

Sizes and powers of the BCIPS test with two known factors ($m=2$) in which factors are serially uncorrelated but idiosyncratic errors are serially correlated with an Intercept only

	$\kappa = 1$					$\kappa = 2$					$\kappa = 3$				
$T \backslash N$	*20*	*30*	*50*	*100*	*200*	*20*	*30*	*50*	*100*	*200*	*20*	*30*	*50*	*100*	*200*
$\alpha_{iy,1}, \alpha_{iy,2}, \alpha_{ix,1}, \alpha_{ixx,2} \sim \text{i.i.d.}U(1,2)$															
Size: $BCIPS(\hat{p}, \kappa)$, positive correlation in idiosyncratic errors															
50	0.050	0.042	0.038	0.038	0.034	0.034	0.036	0.034	0.021	0.037	0.032	0.035	0.017	0.022	0.015
70	0.050	0.041	0.045	0.045	0.040	0.040	0.040	0.040	0.031	0.039	0.037	0.034	0.034	0.024	0.029
100	0.049	0.044	0.047	0.044	0.038	0.043	0.033	0.048	0.037	0.039	0.050	0.024	0.032	0.028	0.041
200	0.051	0.053	0.054	0.053	0.052	0.052	0.043	0.049	0.039	0.048	0.050	0.037	0.031	0.035	0.032
Size: $BCIPS(\hat{p}, \kappa)$, negative correlation in idiosyncratic errors															
50	0.059	0.055	0.055	0.059	0.054	0.054	0.059	0.064	0.050	0.070	0.065	0.069	0.053	0.085	0.094
70	0.049	0.046	0.054	0.060	0.059	0.050	0.059	0.058	0.047	0.063	0.057	0.059	0.059	0.065	0.072
100	0.047	0.054	0.048	0.054	0.050	0.046	0.044	0.059	0.054	0.051	0.062	0.038	0.061	0.055	0.065
200	0.049	0.053	0.055	0.054	0.055	0.059	0.046	0.054	0.043	0.058	0.055	0.049	0.039	0.046	0.047
Power: $BCIPS(\hat{p}, \kappa)$, positive correlation in idiosyncratic errors															
50	0.275	0.321	0.348	0.362	0.394	0.305	0.354	0.386	0.387	0.404	0.287	0.368	0.353	0.434	0.476
70	0.360	0.526	0.588	0.671	0.776	0.446	0.669	0.695	0.688	0.757	0.474	0.751	0.813	0.837	0.901
100	0.766	0.800	0.938	0.986	0.994	0.907	0.916	0.977	0.994	0.998	0.967	0.960	0.995	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Power: $BCIPS(\hat{p}, \kappa)$, negative correlation in idiosyncratic errors															
50	0.296	0.335	0.344	0.353	0.359	0.388	0.439	0.475	0.484	0.492	0.511	0.591	0.622	0.722	0.793
70	0.351	0.599	0.625	0.716	0.830	0.511	0.795	0.829	0.838	0.916	0.639	0.921	0.954	0.977	0.989
100	0.812	0.860	0.975	0.998	1.000	0.946	0.972	0.999	1.000	1.000	0.987	0.997	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Same as those in Table 1.

TABLE 4

Sizes and powers of the BCIPS test with two known factors ($m = 2$) in which factors are serially uncorrelated but idiosyncratic errors are serially correlated with an Intercept only

	$\kappa = 1$					$\kappa = 2$					$\kappa = 3$				
$T \backslash N$	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200
$\alpha_{iy,1}, -\alpha_{iy,2} \sim$ i.i.d.$U(10,20)$, $-\alpha_{ix1}, \alpha_{ix2} \sim$ i.i.d.$U(3,5)$															
Size: $BCIPS(\hat{p},\kappa)$, positive correlation in idiosyncratic errors															
50	0.042	0.049	0.043	0.030	0.045	0.036	0.039	0.027	0.031	0.026	0.037	0.026	0.025	0.019	0.016
70	0.043	0.043	0.039	0.045	0.030	0.039	0.040	0.029	0.034	0.038	0.041	0.033	0.038	0.027	0.021
100	0.042	0.045	0.044	0.046	0.041	0.041	0.042	0.041	0.040	0.042	0.048	0.034	0.039	0.033	0.038
200	0.051	0.054	0.046	0.048	0.052	0.048	0.047	0.040	0.042	0.043	0.041	0.044	0.047	0.038	0.035
Size: $BCIPS(\hat{p},\kappa)$, negative correlation in idiosyncratic errors															
50	0.054	0.055	0.055	0.053	0.073	0.055	0.062	0.059	0.062	0.058	0.069	0.062	0.070	0.073	0.081
70	0.049	0.046	0.048	0.057	0.054	0.053	0.060	0.048	0.053	0.064	0.058	0.059	0.075	0.062	0.071
100	0.046	0.048	0.052	0.054	0.050	0.047	0.052	0.055	0.055	0.061	0.063	0.055	0.068	0.062	0.073
200	0.046	0.051	0.042	0.047	0.058	0.046	0.053	0.045	0.047	0.050	0.047	0.054	0.056	0.052	0.058
Power: $BCIPS(\hat{p},\kappa)$, positive correlation in idiosyncratic errors															
50	0.268	0.365	0.391	0.453	0.537	0.311	0.372	0.378	0.412	0.411	0.319	0.405	0.422	0.492	0.563
70	0.419	0.547	0.611	0.809	0.859	0.510	0.712	0.679	0.802	0.861	0.609	0.790	0.840	0.932	0.966
100	0.868	0.819	0.962	0.990	0.999	0.947	0.913	0.986	0.992	1.000	0.985	0.972	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Power: $BCIPS(\hat{p},\kappa)$, negative correlation in idiosyncratic errors															
50	0.235	0.311	0.324	0.367	0.424	0.352	0.404	0.438	0.483	0.499	0.507	0.626	0.690	0.789	0.871
70	0.394	0.553	0.580	0.793	0.879	0.587	0.805	0.830	0.925	0.974	0.803	0.938	0.978	0.996	1.000
100	0.923	0.855	0.980	0.998	1.000	0.991	0.965	0.998	1.000	1.000	0.997	0.996	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

(*continued*)

TABLE 4

(*Continued*)

$T\backslash N$	$\kappa = 1$ 20	30	50	100	200	$\kappa = 2$ 20	30	50	100	200	$\kappa = 3$ 20	30	50	100	200
$\alpha_{iy,1}, \alpha_{iy,2} \sim$ i.i.d.$U(10, 100)$, $\alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U(3, 5)$															
Size: $BCIPS(\hat{p}, \kappa)$, positive correlation in idiosyncratic errors															
50	0.044	0.045	0.035	0.039	0.032	0.039	0.048	0.031	0.036	0.027	0.040	0.031	0.029	0.024	0.017
70	0.043	0.042	0.040	0.038	0.041	0.033	0.041	0.036	0.036	0.046	0.027	0.034	0.035	0.029	0.024
100	0.045	0.041	0.052	0.051	0.036	0.039	0.054	0.044	0.032	0.036	0.033	0.029	0.037	0.028	0.036
200	0.045	0.043	0.045	0.051	0.042	0.043	0.052	0.043	0.044	0.050	0.051	0.030	0.040	0.042	0.027
Size: $BCIPS(\hat{p}, \kappa)$, negative correlation in idiosyncratic errors															
50	0.056	0.064	0.050	0.058	0.057	0.059	0.076	0.056	0.076	0.061	0.079	0.079	0.074	0.074	0.080
70	0.053	0.048	0.042	0.054	0.059	0.049	0.052	0.052	0.053	0.072	0.053	0.063	0.068	0.062	0.067
100	0.047	0.044	0.050	0.057	0.046	0.047	0.060	0.056	0.047	0.049	0.059	0.046	0.063	0.054	0.066
200	0.046	0.039	0.044	0.051	0.044	0.053	0.051	0.044	0.054	0.053	0.054	0.038	0.052	0.051	0.040
Power: $BCIPS(\hat{p}, \kappa)$, positive correlation in idiosyncratic errors															
50	0.168	0.175	0.198	0.224	0.206	0.239	0.231	0.277	0.272	0.298	0.304	0.343	0.390	0.536	0.627
70	0.379	0.355	0.398	0.589	0.733	0.448	0.611	0.743	0.762	0.928	0.588	0.812	0.947	0.970	1.000
100	0.680	0.878	0.987	0.998	1.000	0.920	0.989	1.000	1.000	1.000	0.983	0.998	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Power: $BCIPS(\hat{p}, \kappa)$, negative correlation in idiosyncratic errors															
50	0.099	0.079	0.090	0.064	0.041	0.342	0.280	0.340	0.413	0.499	0.636	0.650	0.778	0.940	0.992
70	0.350	0.352	0.454	0.537	0.772	0.699	0.901	0.982	0.988	1.000	0.909	0.993	1.000	1.000	1.000
100	0.856	0.959	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Same as those in Table 1.

TABLE 5

Sizes of the BCIPS test with two known factors in which factors are serially uncorrelated and κ is unknown – with an Intercept only

	$\kappa = 1$					$\kappa = 2$					$\kappa = 3$				
$T\backslash N$	*20*	*30*	*50*	*100*	*200*	*20*	*30*	*50*	*100*	*200*	*20*	*30*	*50*	*100*	*200*
Size: *BCIPS*$(\hat{p}, \kappa)$, iid in idiosyncratic errors															
50	0.077	0.079	0.070	0.063	0.063	0.094	0.073	0.064	0.074	0.071	0.079	0.060	0.058	0.064	0.062
70	0.079	0.073	0.064	0.068	0.068	0.078	0.067	0.056	0.064	0.068	0.071	0.055	0.057	0.054	0.051
100	0.072	0.068	0.069	0.059	0.067	0.057	0.063	0.058	0.064	0.053	0.050	0.048	0.048	0.054	0.049
200	0.077	0.062	0.070	0.068	0.062	0.070	0.057	0.053	0.051	0.048	0.050	0.042	0.038	0.030	0.027
Size: *BCIPS*$(\hat{p}, \kappa)$, positive correlation in idiosyncratic errors															
50	0.074	0.067	0.067	0.054	0.057	0.093	0.071	0.061	0.067	0.062	0.067	0.047	0.041	0.041	0.032
70	0.080	0.066	0.063	0.064	0.058	0.087	0.071	0.056	0.065	0.058	0.066	0.044	0.037	0.035	0.025
100	0.077	0.072	0.066	0.059	0.060	0.064	0.072	0.060	0.066	0.051	0.052	0.042	0.033	0.034	0.021
200	0.089	0.069	0.073	0.067	0.066	0.076	0.066	0.057	0.060	0.055	0.054	0.043	0.038	0.030	0.019
Size: *BCIPS*$(\hat{p}, \kappa)$, negative correlation in idiosyncratic errors															
50	0.079	0.082	0.077	0.074	0.079	0.100	0.082	0.074	0.089	0.092	0.092	0.081	0.083	0.090	0.099
70	0.079	0.071	0.068	0.071	0.073	0.085	0.077	0.063	0.075	0.081	0.082	0.074	0.068	0.071	0.072
100	0.070	0.070	0.066	0.062	0.064	0.055	0.062	0.064	0.067	0.060	0.067	0.054	0.061	0.063	0.068
200	0.074	0.064	0.066	0.066	0.065	0.066	0.053	0.050	0.053	0.058	0.054	0.047	0.048	0.051	0.048

Notes: Same as those in Table 1. $\alpha_{iy,1}, \alpha_{iy,2}, \alpha_{ix,1}, \alpha_{ix,2} \sim$ i.i.d.$U(1,2)$. Numbers in the table are the sizes of the *BCIPS* statistic in which the frequency parameter (κ) in the Fourier function and the lag order of the model are jointly selected based on the method discussed in section 'A data-driven method of selecting κ and p'. The *BCIPS* statistics is described by equation (28).

the model with an intercept and a linear trend are slightly higher than those from the model with an intercept when residuals are serially correlated, they are reasonable and close to 0.05 for $T \geqslant 100$.

V. Empirical application

The conventional literature examines the validity of long-run PPP by testing the stationarity of real exchange rates based on the model with an intercept only. Smoothing breaks are likely to appear in real exchange rates due to common shocks or long-lived bubbles. It is therefore interesting to apply the *BCIPS* test to re-examine long-run PPP since the test accommodates cross-dependence and smooth breaks in real exchange rates.

Quarterly nominal exchange rates and consumer price indices (CPI) for 30 OECD countries over the 1981Q1–2011Q4 period are downloaded from the IMF's International Financial Statistics (IFS).[16] For euro-zone countries, the dollar-based nominal exchange rates after 1999 were constructed by using the euro-dollar rate and the prefixed exchange rates at 1 January 1999 (Alba and Papell, 2007). The real exchange rate of a country relative to the US is defined as: $q_{it} = \ln(E_{it}) - \ln(P_{it}) + \ln(P^{us}_{it})$, where E is the nominal exchange rate (domestic currency per US dollar) and P and P^{us} are the consumer price indices of a domestic country and the US respectively.

We set $m_{max} = 4$ since Eickmeier (2009) points out that two to six unobserved common factors are sufficient to explain variations in most macroeconomic variables. This suggests that at most three additional $I(1)$ regressors are needed. Additional regressors that are likely to share common factors with real exchange rates include real gross domestic product (gdp), the long-term government bond yield (r^L), the price-dividend yield (pd) and the price of Brent crude oil (p_{oil}). The quarterly data of these four variables are downloaded from Global Financial Data and IFS. The cross-sectional averages of the above variables are defined as follows: $\overline{gdp}_t = \frac{1}{N}\sum_{i=1}^{N}\ln(GDP_{it}/GDPD_{it})$, $\bar{r}^L_t = \frac{1}{N}\sum_{i=1}^{N} 0.25 \times \ln(1 + R^L_{it}/100)$ and $\overline{pd}_t = \frac{1}{N}\sum_{i=1}^{N}\ln(PS_{it}/D_{it})$. The subscripts i and t denote the ith country and the tth period; GDP is the gross domestic product in the domestic currency, and it is seasonally adjusted by X11 if the raw GDP data are not seasonally adjusted; $GDPD$ is the gross domestic product deflator; R^L is the 10-year, long-term government bond yield; and PS and D denote stock prices and dividends, respectively. For the model with an intercept, the additional regressors should also be non-trended. We regress the above four variables with a linear trend, and the non-trended components are computed as the residuals from the above regressions.[17] These four additional variables are not all available for all countries in the panel over the

[16] They are Australia (AUT), Austria (AUS), Belgium (BEL), Canada (CAN), the Czech Republic (CZE), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Hungary (HUN), Iceland (ICE), Ireland (IRE), Italy (ITA), Japan (JAP), Korea (KOR), Luxembourg (LUX), Mexico (MEX), the Netherlands (NET), New Zealand (NEZ), Norway (NOR), Poland (POL), Portugal (POR), Slovakia (SLO), Spain (SPA), Sweden (SWE), Switzerland (SWI), Turkey (TUR), the United Kingdom (UK) and the United States (US).

[17] A downward linear trend is clear in the plots of long-term government bond yields but is less clear in the plots of price-dividend yields, which look likely to exhibit a quadratic trend or no trend. Alternatively, we assume that price-dividend yields have no trend or exhibit a quadratic trend respectively and then re-examine long-run PPP. These changes, however, do not qualitatively affect the results in Table 4. The results are available from the authors upon request.

TABLE 6

The BCIPS and CIPS panel unit-root tests for real exchange rates

$$\Delta q_{it}=c_{i,0}+c_{i,1}\sin(2\pi\kappa t/T)+c_{i,2}\cos(2\pi t/T)+\boldsymbol{c}'_{i,3}\bar{\boldsymbol{z}}_t+\sum_{j=1}^{p}\boldsymbol{c}'_{i,5,j}\Delta\bar{\boldsymbol{Z}}_{t-j}$$
$$+\sum_{j=1}^{p}c_{i6,j}\Delta q_{i,t-1}+b_i q_{i,t-1}+e_{it},\ \text{where } \boldsymbol{z}_{it}=(q_{it},\boldsymbol{x}'_{it})'.$$

Included $\boldsymbol{x}_{it}$	$(\hat{p},\hat{\kappa})$	$[N,T]$	*CD*	*BCIPS*	*CIPS*
$\hat{p}$ is determined by the SBC rule in equation (39), $m=1$					
No	(1,1)	[29,124]	116.7*	−3.390**	−2.108
m=2					
$\overline{gdp}$	(1,1)	[19,124]	83.8*	−3.757**	−2.867**
p_{oil}	(1,1)	[29,124]	116.7*	−3.228*	−2.116
$\bar{r}^L$	(1,1)	[20,124]	98.5*	−3.331*	−2.658**
$\overline{pd}$	(1,1)	[16,124]	74.3*	−3.245	−2.752**
m=3					
$\overline{gdp},p_{oil}$	(1,1)	[19,124]	83.8*	−3.510*	−2.993**
$p_{oil},\bar{r}^L$	(1,1)	[20,124]	98.5*	−3.048	−2.709*
$\bar{r}^L,\overline{gdp}$	(1,1)	[17,124]	82.0*	−3.701**	−2.936**
$\overline{pd},\overline{gdp}$	(1,1)	[15,124]	68.6*	−3.770**	−3.418**
$\overline{pd},p_{oil}$	(1,1)	[16,124]	74.3*	−3.015	−2.749*
$\overline{pd},\bar{r}^L$	(1,1)	[15,124]	68.6*	−3.206	−2.781*
m=4					
$\overline{gdp},p_{oil},\bar{r}^L$	(1,1)	[17,124]	82.0*	−3.458	−2.918*
$\overline{pd},p_{oil},\bar{r}^L$	(1,1)	[15,124]	68.6*	−2.974	−2.713
$\overline{gdp},\overline{pd},\bar{r}^L$	(1,1)	[15,124]	68.6*	−3.495	−3.309**
$\overline{gdp},p_{oil},\overline{pd}$	(1,1)	[15,124]	68.6*	−3.775**	−3.367**

Notes: m is the number of factors in the model. *CD* is the cross-sectional dependence test of Pesaran (2004). '**' indicates significance at the 1% level and '*' indicates significance at the 5% level . $\hat{\kappa}$ and $\hat{p}$ are jointly determined based on the rule of minimum sum of square described in section 'A data-driven method of selecting κ and p'.

period of 1981–2011. There are 19 series for *gdp*, 20 series for r^L, 16 series for *pd* and 29 series for q_{it}.

The *CIPS* and *BCIPS* statistics are applied to examine the joint unit-root hypothesis if the cross-sectional dependence test provided by Pesaran (2004) rejects the hypothesis of no cross dependence. The common lag order in the *CIPS* test is determined based on the *SBC* rule in equation (39). The common lag order and frequency parameter in the *BCIPS* test are jointly determined as discussed in section 'A data-driven method of selecting κ and p'.

We start from the single-factor case which includes no additional regressors in the *CADF* and *BCADF* regressions. One, two and three additional regressors are respectively included in the *CADF* and *BCADF* regressions for the two, three and four factors cases. The sets of additional regressors for the two-, three- and four-factor cases are $\{\overline{gdp},p_{oil},\overline{pd},\bar{r}^L\}$, $\{(\overline{gdp},p_{oil}),(p_{oil},\bar{r}^L),(\overline{gdp},\bar{r}^L),(\overline{pd},\overline{gdp}),(\overline{pd},p_{oil}),(\overline{pd},\bar{r}^L)\}$ and $\{(\overline{gdp},\bar{r}^L,p_{oil}),(\overline{pd},\bar{r}^L,p_{oil}),(\overline{gdp},\overline{pd},\bar{r}^L),(\overline{gdp},p_{oil},\overline{pd})\}$ respectively.

Table 6 indicates that the *CIPS* and *BCIPS* tests reject the joint unit-root hypothesis at the 5% level for 12 out of 15 and 8 out of 15 cases respectively and there are six cases in which the *CIPS* instead of the *BCIPS* test rejects the unit-root hypothesis. Moreover, for those six cases, the estimated κ is 1. The above results are consistent with the simulation results in Table 2, which indicate that the *CIPS* test may have serious oversize distortions for $\kappa = 1$ and T close to 100 when smooth Fourier breaks exist. Besides, the evidence of rejecting the unit-root hypothesis based on the *BCIPS* test declines with the number of factors.

Next, we apply the information criteria, IC_{p1}, IC_{p2}, and IC_{p3}, proposed by Bai and Ng (2002) to estimate the number of unknown factors, m, in the panel of real exchange rates. The maximum number of factors is set to 4. We first remove smooth breaks in the deterministic term from the data.[18] Let $\boldsymbol{M}_{\Delta\boldsymbol{D}} = \boldsymbol{I} - \ddot{\boldsymbol{\Upsilon}}(\ddot{\boldsymbol{\Upsilon}}'\ddot{\boldsymbol{\Upsilon}})^{-1}\ddot{\boldsymbol{\Upsilon}}'$, where $\ddot{\boldsymbol{\Upsilon}} = (\Delta\boldsymbol{\Upsilon}_1, \Delta\boldsymbol{\Upsilon}_2)$, $\Delta\boldsymbol{\Upsilon}_1 = (\Delta\sin(2\pi\kappa 1/T),\ldots, \Delta\sin(2\pi\kappa T/T))'$, and $\Delta\boldsymbol{\Upsilon}_2 = (\Delta\cos(2\pi\kappa 1/T),\ldots, \Delta\cos(2\pi\kappa T/T))'$. Following Bai and Ng (2004), we transform $\Delta\boldsymbol{q}_i$ to obtain $\Delta\ddot{\boldsymbol{q}}_i$ with different values of κ: $\Delta\ddot{\boldsymbol{q}}_i = \boldsymbol{M}_{\Delta\boldsymbol{D}}\Delta\boldsymbol{q}_i$, where $\Delta\boldsymbol{q}_i = (\Delta q_{i1},\ldots, \Delta q_{iT})'$. Then we apply the *IC* criteria to $\Delta\ddot{\boldsymbol{q}}_i$ for $i = 1,\ldots, N$. Based on the above three *IC* criteria, the estimated number of factors is four regardless of the values of κ. Given that the estimated number of factors is four, the *BCIPS* test with three additional regressors reveals little evidence to reject the unit-root hypothesis. Similar results are observed if the automatic lag-length selection rule employed by Bai and Ng (2004) is applied: $\hat{p} = int[4(min\{N, T\}/100)^{0.25}]$, as indicated by Table B14 in Appendix S2. We, therefore, conclude that there is little evidence to support long-run PPP.

VI. Conclusion

This paper develops a simple panel unit-root test, *BCIPS*, that accommodates cross-sectional dependence among variables and smooth structural changes in deterministic components. The data generation process is generalized to allow for multiple factors. It first shows that the asymptotic null distribution of the individual *BCADF* statistic does not depend on nuisance parameters when N approaches infinity under a fixed T or when both T and N go to infinity. The limiting distribution of the (truncated) *BCIPS* statistic is shown to exist and its critical values are tabulated. Finite-sample properties of the *BCIPS* test are then investigated by Monte-Carlo simulations. The simulation results support that the limiting distribution of our proposed statistic does not depend on nuisance parameters, that the sizes (powers) of the statistic are generally good as long as $T \geqslant 50$ ($T \geqslant 100$), and that the powers of the test increase with κ. The above results indicate that the application of the *BCIPS* test is suggested for $T \geqslant 100$ when smoothing breaks exist in the data. It is fair to say that the *BCIPS* test complements the panel unit-root tests using dummy variables. Finally, the *BCIPS* test is applied to examine long-run PPP, and the results reveal little evidence to support it.

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[18] This is because the matrix format of the null process is (equation (40) in Appendix S1): $\Delta\boldsymbol{y}_i = \Delta\boldsymbol{D}\boldsymbol{\alpha}_{iy} + \boldsymbol{F}\boldsymbol{\gamma}_{iy} + \boldsymbol{\varepsilon}_{iy}$, $i = 1,\ldots, N$. Hence, we remove $\Delta\boldsymbol{D}$ before estimating the factors.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix S1: Mathematical Proofs.
Appendix S2: Critical Values and Supplemental Tables.